Unit 4: Measures of Central Tendency & Dispersion
IB Math AA SL

Answer all 15 questions. Show all working. For Paper 1 questions, use analytical algebraic methods. For Paper 2 questions, use your graphic display calculator (GDC) efficiently.

1. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
   The number of goals scored by a football team in their last 7 matches are:

   $$1, \quad 3, \quad 0, \quad 2, \quad 2, \quad 5, \quad 1$$

   Find the exact values of:

   (a) the mode.

   (b) the median.

   (c) the mean.

2. **[Paper 2 Style, Calculator Required, Easy, 4 marks]**
   The shoe sizes of 25 students in a mathematics class are recorded in the frequency table below.

Shoe Size	6	7	8	9	10
Frequency	3	5	9	6	2

   Use your graphic display calculator to find:

   (a) the mean shoe size.

   (b) the standard deviation.

   (c) the variance.

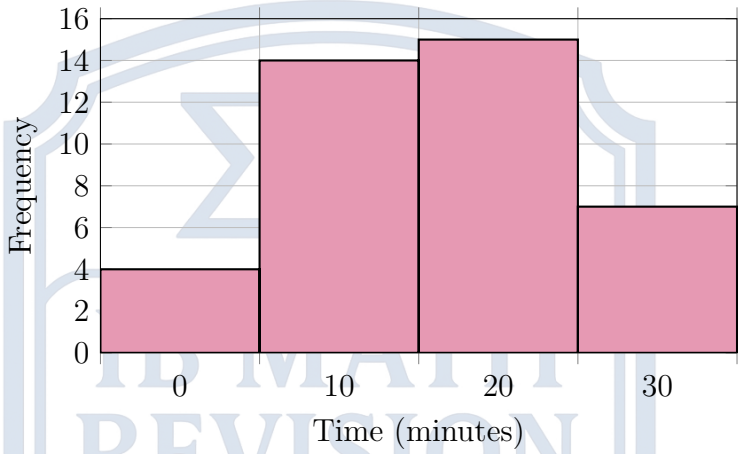
3. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
A dataset of exam scores has a mean of 42 and a standard deviation of 5. The teacher decides to adjust the grades by multiplying every score by 2, and then adding 10 to each score. Find the exact value of:

   (a) the new mean.

   (b) the new standard deviation.

   (c) the new variance.

4. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
The times taken (in minutes) for 40 customers to receive their food at a restaurant are displayed in the histogram below.



   (a) Write down the mid-interval value for the class $20 \leq t < 30$.

   (b) Using the mid-interval values, use your GDC to estimate the mean and the standard deviation of the waiting times.

5. **[Paper 1 Style, Non-Calculator, Medium, 4 marks]**
The mean weight of 5 apples is 120 g. A 6^{th} apple is added to the group, and the new mean weight of the 6 apples becomes 125 g. Calculate the exact weight of the 6^{th} apple.

6. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
A botanist measures the heights, x in cm, of 30 plants. She calculates the following summary statistics:

$$\sum x = 630 \quad \text{and} \quad \sum x^2 = 14220$$

(a) Calculate the exact mean height of the plants.

(b) Using the formula $\text{Var}(X) = \frac{\sum x^2}{n} - \mu^2$, calculate the variance of the plant heights.

(c) Hence, find the standard deviation, correct to three significant figures.

7. **[Paper 1 Style, Non-Calculator, Medium, 5 marks]**
The box-and-whisker diagram below shows the salaries, S in thousands of dollars, of employees at Company A.

(a) Write down the interquartile range (IQR) of the salaries.

(b) Company B decides to match Company A's salary structure by taking their own employees' salaries, multiplying them by 1.5, and then adding 5 thousand dollars. If the IQR of Company B's new salaries matches Company A exactly, find the original IQR of Company B's salaries before the adjustment.

8. **[Paper 2 Style, Calculator Required, Medium, 4 marks]**
There are two mathematics classes, Class P and Class Q, who take the same examination. Class P has 24 students and a mean score of 68%. Class Q has 16 students and a mean score of 75%. Find the overall mean score of all 40 students combined. Give your answer to one decimal place.

9. **[Paper 1 Style, Non-Calculator, Hard, 5 marks]**
A dataset consists of five discrete integers:

$$-2, \quad 1, \quad a, \quad b, \quad 7$$

The mode of the dataset is 1 and the mean is 3.

(a) Deduce the values of a and b.

(b) Calculate the exact variance of the dataset.

10. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
The number of pets owned by families in an apartment building is recorded in the table below.

Number of pets (x)	1	2	3	4
Frequency (f)	2	k	4	1

The mean number of pets per family is exactly 2.4.

(a) By setting up an equation for the mean, find the exact integer value of k.

(b) Use your graphic display calculator to find the standard deviation of the number of pets.

11. **[Paper 1 Style, Non-Calculator, Hard, 5 marks]**
The temperature of a chemical reaction is measured in degrees Celsius (°C). Over 20 trials, the mean temperature is 30°C and the variance is 25. The scientist converts all the temperatures to degrees Fahrenheit (°F) using the formula:

$$F = \frac{9}{5}C + 32$$

Find the exact values of:

(a) the mean temperature in °F.

(b) the standard deviation of the temperatures in °F.

12. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
A dataset of 15 values has a mean of 45 and a standard deviation of 12. During a review of the data, it is discovered that one of the values, 80, was recorded in error and must be removed from the dataset.

(a) Find the new mean of the remaining 14 values.

(b) Calculate $\sum x^2$ for the original dataset of 15 values.

(c) Hence, find the standard deviation of the new dataset with the anomaly removed.

13. **[Paper 1 Style, Non-Calculator, Very Hard, 6 marks]**
A dataset of n numbers has a mean of μ and a standard deviation of σ. A new dataset is created by applying a linear transformation to each value in the original dataset: each value is increased by 3, and then the result is multiplied by 2. The mean of this newly transformed dataset is 30, and its variance is 36. By forming and solving algebraic equations, find the exact values of μ and σ.

14. **[Paper 2 Style, Calculator Required, Very Hard, 7 marks]**
Two different groups of students took a memory test. Group A consists of 20 students. Their mean score is 15 and the standard deviation is 3. Group B consists of 30 students. Their mean score is 20 and the standard deviation is 4. The two groups are combined into a single group of 50 students.

(a) Find the combined mean score.

(b) By calculating the total sum of squares ($\sum x^2$) for each group separately, find the overall standard deviation of the combined group.

15. **[Paper 1 Style, Non-Calculator, Very Hard, 7 marks]**
The mean of three numbers, a, b, and c, is exactly 8. The variance of these three numbers is 10.

(a) Using the formula for variance, prove that $a^2 + b^2 + c^2 = 222$.

(b) Given that $c = 10$, find the mean and the variance of the remaining two numbers, a and b.