

IB MATHEMATICS AA HL

AHL TOPIC 2 PRACTICE

Polynomials & Rational Functions

Instructions to Candidates

- This practice paper contains **20** questions progressing from Easy to Very Hard.
- Each question indicates whether it is styled for **Paper 1 (No Calculator)** or **Paper 2 (Calculator Allowed)**.
- The paper tests syllabus topics AHL 2.12 and 2.13: Factor and remainder theorems, sum and product of roots, complex roots, solving inequalities, and rational functions with oblique asymptotes.
- Answer all questions, showing all your working clearly.
- Total marks available: **86**.

Difficulty Progression

- **SECTION A (Easy):** Basic polynomial remainder theorem, identifying vertical/horizontal asymptotes, sum and product of cubic roots, and simple graphical inequalities.
- **SECTION B (Medium):** Finding unknown polynomial coefficients, algebraic division for oblique asymptotes, solving rational inequalities analytically, and complex conjugate root pairs.
- **SECTION C (Hard):** Vieta's formulas for quartics, finding the exact range of a rational function using discriminants, polynomial roots forming arithmetic progressions, and algebraic proofs.

SECTION A: EASY (Fundamentals)

Question 1 (3 Marks) — Paper 1 (No Calculator Allowed)

Let the polynomial be $P(x) = 2x^3 - x^2 + kx - 4$. When $P(x)$ is divided by $(x - 2)$, the remainder is 10. Find the value of the constant k .

Question 2 (2 Marks) — Paper 1 (No Calculator Allowed)

Consider the rational function $f(x) = \frac{3x-2}{x+4}$. Write down the equations of its vertical and horizontal asymptotes.

Question 3 (4 Marks) — Paper 1 (No Calculator Allowed)

A cubic polynomial equation is given by $2x^3 - 7x^2 + 4x - 5 = 0$, and its roots are α, β , and γ . Find the exact value of:

(a) $\alpha + \beta + \gamma$ [2 marks]

(b) $\alpha\beta\gamma$ [2 marks]

Question 4 (3 Marks) — Paper 2 (Calculator Allowed)

Solve the polynomial inequality $x^3 - 4x^2 + x + 6 < 0$.

Question 5 (4 Marks) — Paper 1 (No Calculator Allowed)

Show that $(x - 2)$ is a factor of $P(x) = x^3 - 7x + 6$, and hence fully factorise the polynomial $P(x)$ into three linear factors.

Question 6 (3 Marks) — Paper 1 (No Calculator Allowed)

Find the coordinates of the x -intercepts and the y -intercept of the graph of $y = \frac{x^2-4}{x-1}$.

Question 7 (3 Marks) — Paper 1 (No Calculator Allowed)

Expand and simplify $(2x - 1)^3$, giving your answer in the form $ax^3 + bx^2 + cx + d$.

SECTION B: MEDIUM (Application & Algebraic Methods)**Question 8 (6 Marks) — Paper 1 (No Calculator Allowed)**

The polynomial $P(x) = x^3 + ax^2 + bx + 6$ has exactly two linear factors, $(x + 1)$ and $(x - 3)$. Find the values of the constants a and b .

Question 9 (4 Marks) — Paper 1 (No Calculator Allowed)

Find the equation of the oblique (slant) asymptote of the rational function:

$$y = \frac{3x^2 - 2x + 1}{x - 1}$$

Question 10 (6 Marks) — Paper 1 (No Calculator Allowed)

A cubic equation $z^3 + pz^2 + qz + 15 = 0$, where p and q are real constants, has $1 + 2i$ as one of its roots. Find the exact values of p and q .

Question 11 (5 Marks) — Paper 1 (No Calculator Allowed)

Solve the rational inequality analytically:

$$\frac{x - 4}{x + 1} \leq 3$$

Question 12 (4 Marks) — Paper 1 (No Calculator Allowed)

A cubic equation $x^3 - 3x^2 + 2x - 5 = 0$ has roots α, β , and γ . By using the properties of the sum and product of polynomial roots, find the exact value of $\alpha^2 + \beta^2 + \gamma^2$.

Question 13 (4 Marks) — Paper 1 (No Calculator Allowed)

Consider the rational function $f(x) = \frac{x^2 - 9}{x^2 - x - 6}$. Find the equation of the vertical asymptote and the coordinates of the "hole" (removable singularity) on the graph.

Question 14 (4 Marks) — Paper 2 (Calculator Allowed)

Use your graphic display calculator to find the coordinates of all points of intersection between the polynomial $f(x) = x^4 - 2x^2 + 1$ and the linear function $g(x) = 2x + 3$.

SECTION C: HARD / VERY HARD (Synthesis & Proof)**Question 15 (7 Marks) — Paper 2 (Calculator Allowed)**

Consider the rational function $y = \frac{x^2+2x+5}{x+1}$ for $x \neq -1$. By rearranging the equation into a quadratic in x and analyzing its discriminant, find the exact range of the function.

Question 16 (8 Marks) — Paper 1 (No Calculator Allowed)

The polynomial equation $x^3 - 6x^2 + 13x - 10 = 0$ has three roots that form an arithmetic progression. Find all three roots of the equation, leaving complex roots in the form $a + bi$.

Question 17 (5 Marks) — Paper 1 (No Calculator Allowed)

Sketch the graph of the rational function $y = \frac{2x^2}{x^2-4}$. Clearly label all intercepts and equations of all vertical and horizontal asymptotes.

Question 18 (4 Marks) — Paper 1 (No Calculator Allowed)

Prove the Sophie Germain polynomial identity algebraically:

$$x^4 + 4y^4 \equiv (x^2 - 2xy + 2y^2)(x^2 + 2xy + 2y^2)$$

Question 19 (3 Marks) — Paper 2 (Calculator Allowed)

Determine the number of distinct real solutions to the equation:

$$x^3 - 2x^2 + x - 1 = \frac{3}{x}$$

Question 20 (4 Marks) — Paper 1 (No Calculator Allowed)

A quartic equation $x^4 - 2x^3 + x^2 - 4x + 2 = 0$ has roots α, β, γ , and δ . Find the exact value of the expression:

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$$