Unit 2: Graphing Functions & Key Features (Technology)
IB Math AA SL

Answer all questions. Show all working where appropriate. Use your graphic display calculator (GDC) to efficiently locate intercepts, asymptotes, vertices, and intersections.

1. **[Paper 2 Style, Short Answer, Easy, 4 marks]**
   Consider the quadratic function $f(x) = 2x^2 - 5x - 12$.

   (a) Use your graphic display calculator to find the coordinates of the x-intercepts.

   (b) Find the coordinates of the vertex of the graph of f.

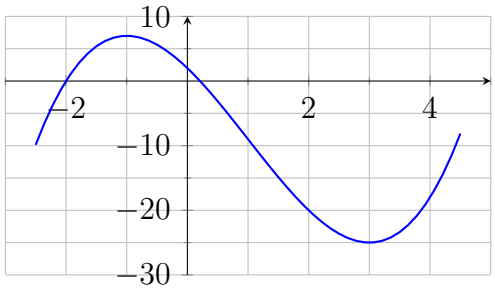
2. **[Paper 2 Style, Short Answer, Easy, 4 marks]**
   Let the function $g(x)$ be defined by $g(x) = \frac{3x+1}{x-2}$ for $x \neq 2$.

   (a) Write down the equation of the vertical asymptote.

   (b) Find the equation of the horizontal asymptote.

   (c) Find the coordinates of the y-intercept.

3. **[Paper 2 Style, Short Answer, Easy, 4 marks]**
   The function $h(x) = x^3 - 3x^2 - 9x + 2$ is sketched below.



   Using the graphing features of your GDC, find:

   (a) the coordinates of the local maximum point.

   (b) the coordinates of the local minimum point.

4. **[Paper 2 Style, Short Answer, Easy, 5 marks]**
Let $f(x) = 4 - 2e^x$.

   (a) Find the exact coordinates of the y-intercept of the graph of f.

   (b) Use your GDC to find the x-intercept, giving your answer to three significant figures.

   (c) Write down the equation of the horizontal asymptote.

5. **[Paper 2 Style, Short Answer, Medium, 5 marks]**
Consider the curves $f(x) = 5 - x^2$ and $g(x) = e^{0.5x}$.

   (a) Graph both functions on your graphic display calculator for $-4 \leq x \leq 4$. Sketch the result on your paper, clearly marking the points of intersection.

   (b) Find the x-coordinates of their points of intersection to three significant figures.

6. **[Paper 2 Style, Short Answer, Medium, 6 marks]**
The graph of the rational function $f(x) = \frac{ax-4}{3x+b}$ has a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = -1$.

   (a) Deduce the value of a and the value of b.

   (b) Find the coordinates of the x-intercept of the graph of f.

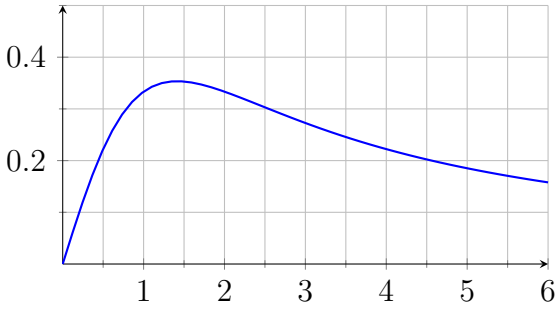
7. **[Paper 2 Style, Longer Question, Medium, 6 marks]**
A parabolic arch over a river can be modelled by the function $h(x) = 4 - \frac{x^2}{8}$, where h is the height of the arch above the water level, and x is the horizontal distance from the centre of the arch. All measurements are in metres.

   (a) Write down the maximum height of the bridge above the water.

   (b) Use your GDC to find the x-intercepts of the graph.

   (c) Hence, determine whether the bridge is wide enough to completely span a river that is 11 m wide. Justify your answer.

8. **[Paper 2 Style, Short Answer, Medium, 5 marks]**
The following diagram shows part of the graph of $y = \frac{x}{x^2+2}$ for $x \geq 0$.



(a) Use your GDC to find the coordinates of the local maximum point, giving your values to three significant figures.

(b) Determine the equation of the horizontal asymptote as $x \to \infty$.

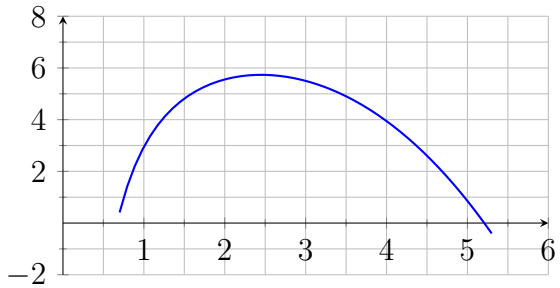
9. **[Paper 2 Style, Short Answer, Medium, 4 marks]**
Let $f(x) = \frac{4}{x} + x^2 - 4$ for $x > 0$. Use your graphic display calculator's minimum finding feature to calculate the exact coordinates of the local minimum point of the graph of $f(x)$.

10. **[Paper 2 Style, Short Answer, Hard, 5 marks]**
Let $f(x) = \sin(e^x)$ for $0 \leq x \leq 1.5$. The angle x is measured in radians.

(a) Sketch the graph of f on the given domain.

(b) Find the exact x-intercept of the graph of f.

11. **[Paper 2 Style, Longer Question, Hard, 6 marks]**
The diagram below shows a part of the graph of the function $f(x) = 9 - \frac{1}{18}x^3 - \frac{6}{x}$ for $x > 0$.



(a) Using your GDC, find the coordinates of the two points where the curve crosses the x-axis.

(b) Find the coordinates of the local maximum point.

12. **[Paper 2 Style, Longer Question, Hard, 6 marks]**
Consider the function $f(x) = \ln(x + 2)$ for $x > -2$.

(a) Write down the equation of the vertical asymptote of the graph of f.

(b) The graph of $y = f(x)$ intersects the graph of its inverse function, $y = f^{-1}(x)$, exactly twice. Use your GDC to find the coordinates of these two intersection points to three significant figures.

13. **[Paper 2 Style, Longer Question, Hard, 7 marks]**
The line L_1 with equation $y = 8x - 4$ intersects the curve C with equation $y = -x^3 + x^2 + 10x + 8$.

(a) Using your GDC, find the coordinates of the three points of intersection between L_1 and C.

(b) Graph L_1 and C on your GDC. Sketch the result, clearly shading the region bounded completely between the two graphs in the first quadrant.

14. **[Paper 2 Style, Extended Question, Very Hard, 8 marks]**
Let $f(x) = \left|\frac{6x-3}{2x-9}\right|$ for $x \neq 4.5$.

(a) Sketch the graph of $f(x)$, explicitly labelling the coordinates of the x-intercept and y-intercept.

(b) Write down the equations of the vertical and horizontal asymptotes.

(c) The horizontal line $y = k$ intersects the graph of $f(x)$ exactly once. Find the two possible values of k.

15. **[Paper 2 Style, Extended Question, Very Hard, 8 marks]**
The functions f and g are defined by $f(x) = \ln(2x - 9)$ for $x > 4.5$, and $g(x) = 2\ln x - \ln d$ for $x > 0$, where $d \in \mathbb{R}^+$.

(a) State the equation of the vertical asymptote to the graph of $y = g(x)$.

(b) The graphs of $y = f(x)$ and $y = g(x)$ intersect at two distinct points. Show that, at the points of intersection, $x^2 - 2dx + 9d = 0$.

(c) Hence show that $d^2 - 9d > 0$.

(d) Find the range of possible values for d.