Unit 2: Transformations of Graphs
IB Math AA SL

Answer all questions. Show all working where appropriate. For Paper 1 questions, you must use analytical algebraic methods. For Paper 2 questions, use your graphic display calculator (GDC) to efficiently locate roots, vertices, and intersections.

1. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
   The point $P(4, -2)$ lies on the curve with equation $y = f(x)$. State the coordinates of the image of point P on the curves with the following equations:

   (a) $y = f(x) - 3$

   (b) $y = f(x - 2)$

   (c) $y = 2f(x)$

   (d) $y = f(-x)$

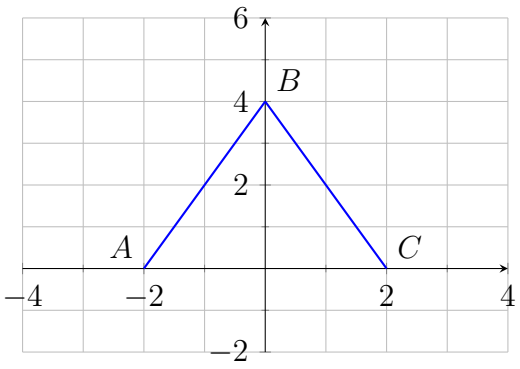
2. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
   Let $f(x) = x^2 - 4$. The graph of $g(x)$ is obtained by translating the graph of $f(x)$ by the vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

   (a) Find an expression for $g(x)$, fully expanded.

   (b) Use your graphic display calculator to sketch both $f(x)$ and $g(x)$ on the same axes.

   (c) Find the exact coordinates of the vertex of the graph of $g(x)$.

3. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
The diagram below shows the graph of $y = f(x)$. The graph intersects the coordinate axes at the marked points $A(-2, 0)$, $B(0, 4)$, and $C(2, 0)$.



In separate diagrams on your paper, sketch the curves with equations:

(a) $y = -f(x)$

(b) $y = f(2x)$

On each diagram, clearly label the coordinates of the images of points A, B, and C.

4. **[Paper 2 Style, Calculator Required, Easy, 4 marks]**
Consider the function $h(x) = e^x$. The graph of $k(x)$ is obtained by a vertical stretch of scale factor 3, followed by a translation of $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$.

(a) Write down an expression for $k(x)$.

(b) Use your GDC to find the exact x-intercept of the graph of $k(x)$, giving your answer to three significant figures.

5. **[Paper 1 Style, Non-Calculator, Medium, 5 marks]**
Given that $f(x) = 3x^2 - 2x$, find an expression for $g(x)$, where $g(x)$ is obtained by applying the following sequence of transformations to $f(x)$:

- a translation by the vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$,
- followed by a vertical stretch of scale factor 4.

Give your final answer in the form $g(x) = ax^2 + bx + c$.

6. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
Let $f(x) = \frac{3x}{x-2}$ for $x \neq 2$.

   (a) State the equations of the vertical and horizontal asymptotes of the graph of $y = f(x)$.

   (b) The function g is defined as $g(x) = f(x+4) - 1$. Determine the equations of the asymptotes of $g(x)$.

   (c) Use your GDC to find the coordinates where the graph of $g(x)$ intersects the x-axis.

7. **[Paper 1 Style, Non-Calculator, Medium, 6 marks]**
Let $f(x) = \frac{1}{x}$. The function $g(x)$ is obtained by translating $f(x)$ right by 3 units and up by 2 units.

   (a) Write down an expression for $g(x)$.

   (b) Show algebraically that $g(x)$ can be written in the form $h(x) = \frac{2x-5}{x-3}$.

   (c) State the domain and range of $h(x)$.

8. **[Paper 2 Style, Calculator Required, Medium, 6 marks]**
Consider the function $f(x) = \sin(x)$ for $0 \leq x \leq 2\pi$. Let $g(x) = 2f(x) + 1$.

   (a) Describe the two transformations that map $f(x)$ onto $g(x)$.

   (b) Sketch both graphs on your GDC.

   (c) Use the intersection feature on your GDC to find the exact coordinates of any points where the graph of $f(x)$ intersects the graph of $g(x)$.

9. **[Paper 1 Style, Non-Calculator, Hard, 6 marks]**
Let $f(x) = 2x^2 - 6x$ for $x \in \mathbb{R}$. The graph of a function $g(x)$ is obtained from the graph of $f(x)$ by a reflection in the y-axis, followed by a horizontal stretch with scale factor $\frac{1}{2}$.

   (a) Find an expression for $g(x)$.

   (b) Express $g(x)$ in the vertex form $a(x-h)^2 + k$.

   (c) Write down the coordinates of the minimum point of $g(x)$.

10. **[Paper 2 Style, Calculator Required, Hard, 5 marks]**
Let $f(x) = \ln(x)$ for $x > 0$. The graph of $g(x)$ is defined as $g(x) = -\ln(x+2) + 4$.

   (a) State the sequence of three transformations that map $f(x)$ onto $g(x)$.

   (b) Use your GDC to find the x-coordinate of the point where the graphs of f and g intersect, giving your answer to three significant figures.

11. **[Paper 1 Style, Non-Calculator, Hard, 6 marks]**
Let $f(x) = \sqrt{x}$. The function $g(x)$ is defined by $g(x) = 3\sqrt{-x-2}$.

   (a) Describe a sequence of three geometric transformations that map the graph of $f(x)$ onto the graph of $g(x)$.

   (b) Find the maximal valid domain of $g(x)$.

   (c) State the range of $g(x)$.

12. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
Consider the cubic function $f(x) = -x^3 + 2x^2 + 5$. Let $g(x) = f(x-2)$.

   (a) Use your GDC to find the coordinates of the local maximum and local minimum points on the graph of $f(x)$.

   (b) Hence, write down the coordinates of the local maximum and local minimum points on the graph of $g(x)$.

   (c) Find the coordinates of the point of intersection between the graphs of $f(x)$ and $g(x)$.

13. **[Paper 1 Style, Non-Calculator, Very Hard, 6 marks]**
Consider the linear function $p(x) = 3x - 4$.

   (a) Find an expression for the inverse function, $p^{-1}(x)$.

   (b) Find an expression for $q(x) = \frac{1}{9}[p(x) + 16]$, fully simplified.

   (c) What can you deduce about the sequence of transformations given by $y = \frac{1}{9}[p(x) + 16]$ relative to the inverse function?

14. **[Paper 2 Style, Calculator Required, Very Hard, 7 marks]**
Let $f(x) = \cos(x)$ for $x \in \mathbb{R}$. The graph of $g(x)$ is given by $g(x) = 3\cos(2x - \pi) + 1$.

   (a) By first rewriting the argument of the cosine function, give a full geometric description of the sequence of four transformations that map $f(x)$ onto $g(x)$.

   (b) Use your GDC to find the smallest positive x-intercept of the graph of $g(x)$, giving your answer to three significant figures.

15. **[Paper 1 Style, Non-Calculator, Very Hard, 7 marks]**
Let $f(x) = \frac{4}{3}(x-5)^3 - 2$ and $g(x) = x^3$, both for $x \in \mathbb{R}$. Give a full geometric description of two *different* individual combinations of transformations that can be combined to obtain the graph of f from the graph of g, given that:

(a) Sequence A: a stretch is to be applied first, followed by a single translation.

(b) Sequence B: a translation is to be applied first, followed by a single stretch.