Unit 2: Equations of Straight Lines
IB Math AA SL

Answer all questions. Show all working where appropriate. Use your graphic display calculator (GDC) to efficiently find intersections and solve linear systems where possible.

1. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
   The equation of a line L_1 is $3x + 4y - 12 = 0$.

   (a) Find the gradient of L_1.

   (b) Find the coordinates of the x-intercept and the y-intercept of L_1.

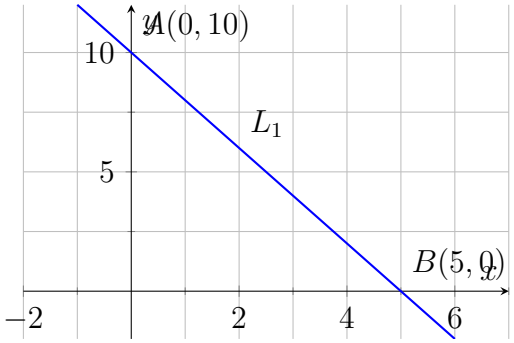
2. **[Paper 1 Style, Short Answer, Easy, 3 marks]**
   The coordinates of point A are $(1, 3)$ and the coordinates of point B are $(4, 9)$. Find the equation of the line passing through A and B. Give your answer in the form $y = mx + c$.

3. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
   A line L_1 has the equation $y = 5x - 2$. A second line, L_2, is parallel to L_1 and passes through the point $(0, 7)$.

   (a) Write down the equation of L_2.

   (b) A third line, L_3, is perpendicular to L_1 and passes through the origin. Find the equation of L_3.

4. **[Paper 2 Style, Short Answer, Easy, 5 marks]**
The diagram below shows the line L_1, which intersects the y-axis at $A(0, 10)$ and the x-axis at $B(5, 0)$.



(a) Find the equation of L_1 in the form $y = mx + c$.

(b) Find the exact length of the line segment $[AB]$.

5. **[Paper 2 Style, Short Answer, Medium, 5 marks]**
Taxi Company A charges a fixed fee of \$4.00 plus \$1.50 per kilometre travelled. Taxi Company B charges a fixed fee of \$2.50 plus \$2.00 per kilometre travelled.

(a) Defining suitable variables, write down the linear equations for the cost of travelling with Company A and Company B.

(b) Use your graphic display calculator to determine the distance for which both taxi companies cost exactly the same amount.

6. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
Point A has coordinates $(x, 4)$ and point B has coordinates $(9, y)$. M is the midpoint of $[AB]$ and has coordinates $(-2, 7)$.

(a) Find the value of x and the value of y.

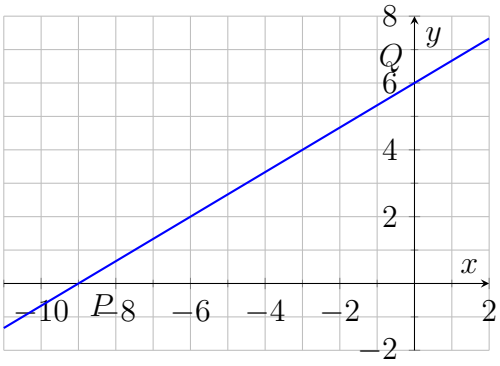
(b) Hence, find the gradient of the line passing through A and B.

7. **[Paper 1 Style, Short Answer, Medium, 6 marks]**
The line L_1 passes through the points $A(-2, 5)$ and $B(4, -7)$.

(a) Find the coordinates of the midpoint of $[AB]$.

(b) Find the equation of the perpendicular bisector of the line segment $[AB]$. Give your answer in the form $ax + by + d = 0$, where a, b, and d are integers.

8. **[Paper 2 Style, Longer Question, Medium, 6 marks]**
The diagram below shows the line L_1 with equation $2x - 3y + 18 = 0$. The line intersects the x-axis at point P and the y-axis at point Q.



(a) Find the coordinates of P and Q.

(b) Calculate the exact area of the triangle OPQ, where O is the origin.

(c) A second line L_2 is parallel to L_1 and passes through $(3, 2)$. Find the equation of L_2 in the form $y = mx + c$.

9. **[Paper 1 Style, Short Answer, Hard, 4 marks]**
The line passing through the points $A(k, 4)$ and $B(6, k)$ has a gradient of 3. Find the exact value of k.

10. **[Paper 2 Style, Short Answer, Hard, 5 marks]**
The line L_1 has the equation $5y - 2x + 1 = 0$. Point A has coordinates $(x, 3)$ and lies on the intersection of L_1 and a second line, L_2. The line L_2 is perpendicular to L_1.

(a) Find the exact value of x.

(b) Write down the equation of L_2, giving your answer in the form $y = mx + c$.

11. **[Paper 2 Style, Short Answer, Hard, 5 marks]**
A straight line L_1 has equation $ax + by + 10 = 0$. The line passes through the points $(1, 4)$ and $(3, -2)$. By setting up a system of linear equations, use your GDC to find the exact values of the constants a and b.

12. **[Paper 1 Style, Longer Question, Hard, 6 marks]**
A quadrilateral has four vertices with coordinates $A(-1,-1)$, $B(2,2)$, $C(4,2)$, and $D(-1,-3)$.

   (a) Show algebraically that the line segments $[AB]$ and $[CD]$ are parallel.

   (b) Find the equation of the line passing through B and C.

   (c) Deduce the geometric shape of the quadrilateral $ABCD$.

13. **[Paper 1 Style, Longer Question, Very Hard, 7 marks]**
The points $P(4,2)$ and $Q(1,8)$ lie on a coordinate plane. The line L_1 is the perpendicular bisector of the line segment $[PQ]$.

   (a) Find the equation of L_1, giving your answer in the form $ax + by + d = 0$, where a, b, and d are integers.

   (b) Point C lies on L_1 such that its x-coordinate is -3.5. Find the length of QC.

14. **[Paper 2 Style, Extended Question, Very Hard, 8 marks]**
The line L_1 has equation $y = x + 2$ and the line L_2 has equation $y = -2x + 14$. The lines intersect at point A.

   (a) Find the coordinates of A.

   (b) Point B lies on L_1 such that its x-coordinate is less than the x-coordinate of A. The distance AB is exactly $\sqrt{18}$ units. Find the coordinates of B.

   (c) The line L_3 passes through point B and is perpendicular to L_1. Find the equation of L_3 and determine where it intersects L_2.

15. **[Paper 1 Style, Extended Question, Very Hard, 8 marks]**
Three points define the vertices of a triangle: $A(-2,-1)$, $B(4,7)$, and $C(8,4)$.

   (a) By calculating the gradients of $[AB]$ and $[BC]$, prove that triangle ABC is a right-angled triangle.

   (b) Calculate the exact lengths of $[AB]$ and $[BC]$.

   (c) Hence, find the exact area of the triangle ABC.