

IB MATHEMATICS AI HL AHL QUESTION BOOKLET

Complex Numbers & Argand Diagrams

Instructions to Candidates

- This extended practice paper contains **15 questions**.
- The paper targets **Advanced Higher Level (AHL)** syllabus components 1.12 and 1.13.
- Note: In Mathematics AI, all papers require a Graphic Display Calculator (GDC). Use it efficiently to solve systems and evaluate complex roots.
- Answer all questions, showing all step-by-step working clearly in the spaces provided.

Difficulty Progression

- **Questions 1 - 5 (Easy):** Basic Cartesian operations, complex conjugates, solving quadratics with complex roots, and converting to Euler/Polar form.
- **Questions 6 - 10 (Medium):** Distances on the Argand diagram, calculating high powers, geometric transformations (rotations/stretch), and conjugate pairs.
- **Questions 11 - 15 (Hard):** Complex synthesis. Solving simultaneous linear equations with complex coefficients, AC circuit impedance, and phasor addition.

SECTION A: EASY (Fundamentals)

Question 1 (4 Marks)

Consider the complex numbers $z_1 = 5 + 2i$ and $z_2 = 3 - 4i$.

Let z_2^* be the complex conjugate of z_2 .

(a) Calculate $z_1 - 2z_2^*$. Give your answer in Cartesian form. [2 marks]

(b) Calculate $\frac{z_1}{z_2}$, giving your answer in the form $a + bi$. [2 marks]

Question 2 (4 Marks)

Solve the quadratic equation $2x^2 - 6x + 5 = 0$.

Give your answers in the exact Cartesian form $a \pm bi$.

Question 3 (4 Marks)

A complex number is given by $z = -3\sqrt{3} + 3i$.

Convert z into exponential (Euler) form, $z = re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$.

Question 4 (4 Marks)

A complex number is given in exponential form as $z = 4e^{i\frac{3\pi}{4}}$.

Express z in exact Cartesian form, $a + bi$.

Question 5 (5 Marks)

Let $w_1 = 6e^{i\frac{\pi}{3}}$ and $w_2 = 2e^{i\frac{\pi}{6}}$.

(a) Find the exact value of $\frac{w_1}{w_2}$ in exponential form. [2 marks]

(b) Hence, express $\frac{w_1}{w_2}$ in exact Cartesian form. [3 marks]

SECTION B: MEDIUM (Application & Modelling)

Question 6 (5 Marks)

Two points, A and B , are plotted on an Argand diagram. Point A represents the complex number $z_A = -2 + 5i$, and point B represents the complex number $z_B = 4 + i$. Calculate the exact geometric distance between point A and point B .

Question 7 (6 Marks)

The complex number $z = 3 - i$ undergoes two geometric transformations in the complex plane. First, it is rotated by $\frac{\pi}{2}$ radians counter-clockwise about the origin. Then, it is stretched away from the origin by a scale factor of 2. Find the new complex number resulting from these transformations, giving your answer in Cartesian form.

Question 8 (6 Marks)

The complex number $z_1 = 2 + 3i$ is known to be a root of the quadratic equation:

$$z^2 + bz + c = 0$$

where b and c are real numbers.

- (a) Write down the second root, z_2 , of this equation. [1 mark]
(b) By using the sum and product of the roots, or otherwise, find the values of b and c . [5 marks]

Question 9 (6 Marks)

Consider the complex number $z = 1 + i$.

- (a) Express z in exponential form. [2 marks]
(b) Hence, calculate z^8 , giving your final answer as an exact real integer. [4 marks]

Question 10 (6 Marks)

In physics, two sinusoidal waveforms of the same frequency can be added together using complex numbers (phasors).

Given $x_1(t) = 5 \cos(10t)$ and $x_2(t) = 12 \sin(10t)$.

By first rewriting $x_2(t)$ as a cosine function with a phase shift, express the sum $X_{total}(t) = x_1(t) + x_2(t)$ in the form $A \cos(10t - \phi)$, where $A > 0$ and ϕ is in radians.

SECTION C: HARD (Synthesis & Proof)**Question 11 (7 Marks)**

Solve the following system of simultaneous linear equations for the complex variables z and w :

$$(2 + i)z + w = 5i$$

$$z - iw = 3$$

Give your final answers for z and w in Cartesian form.

Question 12 (8 Marks)

An electrical engineer is analyzing an AC circuit with two voltage sources connected in series. The voltages (in Volts) are given by:

$$V_1(t) = 15 \cos(60t + 0.5)$$

$$V_2(t) = 25 \cos(60t - 0.7)$$

where t is time in seconds, and the phase angles are in radians.

Express the total voltage $V_T(t) = V_1(t) + V_2(t)$ as a single sinusoidal function in the form $V_T(t) = A \cos(60t + B)$. Give A and B correct to three significant figures.

Question 13 (8 Marks)

A square is drawn on an Argand diagram, centered perfectly at the origin $(0, 0)$. One of the vertices of this square represents the complex number $z_1 = 3 + 4i$.

Find the Cartesian coordinates of the other three vertices of the square.

Question 14 (7 Marks)

In an AC circuit, the total impedance Z_T of two components connected in parallel is given by the formula:

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Given that $Z_1 = 40 + 20i$ ohms and $Z_2 = 30 - 10i$ ohms, calculate Z_T .

Give your final answer for Z_T as an exact real number.

Question 15 (9 Marks)

A three-phase electrical power system generates three separate voltages, which can be modelled by the following complex phasors:

$$V_1 = 230e^{i0}$$

$$V_2 = 230e^{-i\frac{2\pi}{3}}$$

$$V_3 = 230e^{i\frac{2\pi}{3}}$$

- (a) Convert V_2 and V_3 into exact Cartesian form. [5 marks]
(b) Hence, physically prove that the sum of the three voltages is exactly zero ($V_1 + V_2 + V_3 = 0$). [4 marks]

