IB MATHEMATICS AA HL
AHL TOPIC 4 PRACTICE

Conditional Probability, Bayes' Theorem & Venn Diagrams

Instructions to Candidates

- This practice paper contains **15** questions progressing from Easy to Very Hard.
- Each question indicates whether it is styled for **Paper 1 (No Calculator)** or **Paper 2 (Calculator Allowed)**.
- The paper tests syllabus topics SL 4.6 and AHL 4.13: Use of Venn diagrams, conditional probability, mutually exclusive/independent events, and Bayes' Theorem.
- Answer all questions, showing all your working clearly.
- Total marks available: **79**.

Difficulty Progression

- **SECTION A (Easy):** Extracting values from 2-set Venn diagrams, applying conditional probability formulas, basic Bayes' Theorem trees, and independent events.
- **SECTION B (Medium):** Extracting conditional probability from 3-set Venn diagrams, calculating intersections from union formulas, and translating medical Bayes' problems into Venn diagram regions.
- **SECTION C (Hard):** Algebraic Venn diagrams with unknown probabilities, establishing independence algebraically, complex 3-event union formulas ($P(A \cup B \cup C)$), and the Monty Hall problem.

SECTION A: EASY (Fundamentals)

Question 1 (5 Marks) — Paper 1 (No Calculator Allowed)

In a class of 30 students, 19 play tennis (T), 3 play both tennis and volleyball (V), and 6 do not play either sport.

(a) Draw a Venn diagram to represent this information, finding the number of students who play volleyball only. **[3 marks]**

(b) Find the exact probability that a randomly selected student plays tennis, given that they play volleyball. **[2 marks]**

Question 2 (4 Marks) — Paper 1 (No Calculator Allowed)

Events A and B are represented in a universal set. It is given that $P(A) = 0.4$, $P(A|B) = 0.25$, and $P(A \cup B) = 0.55$. Find the exact value of $P(B)$, and hence find $P(B|A)$.

Question 3 (4 Marks) — Paper 2 (Calculator Allowed)

A tech company has two factories that manufacture microchips. Factory 1 produces 60% of the chips, and Factory 2 produces the remaining 40%. It is known that 2% of the chips from Factory 1 are defective, while 5% of the chips from Factory 2 are defective. Given that a randomly selected microchip is found to be defective, find the probability that it was produced by Factory 2.

Question 4 (4 Marks) — Paper 1 (No Calculator Allowed)

Two events X and Y are independent. It is given that $P(X) = 0.5$ and $P(Y) = 0.7$. Find the probability that:

(a) Exactly one of the events occurs. **[2 marks]**

(b) Event X occurs, given that event Y does not occur. **[2 marks]**

Question 5 (5 Marks) — Paper 1 (No Calculator Allowed)

Events A and B have the following conditional probabilities: $P(B|A) = 0.8$ and $P(B|A') = 0.4$. Let the prior probability $P(A) = x$. Given that the posterior probability $P(A|B) = 0.5$, formulate an equation in terms of x and hence find the exact value of x.

SECTION B: MEDIUM (Application & Algebraic Methods)

Question 6 (6 Marks) — Paper 1 (No Calculator Allowed)

In a team of 30 judo players, 13 have won a match by throwing (T), 12 have won by hold-down (H), and 13 have won by points decision (P). 2 players have won matches by all three methods. 5 have won matches by throwing and hold-down. 4 have won matches by hold-down and points decision. 3 have won matches by throwing and points decision. Find the exact probability that a randomly selected player has won a match by throwing (T), given that they have won a match by **exactly one** method.

Question 7 (6 Marks) — Paper 2 (Calculator Allowed)

At a pet shop, G is the event a customer buys a goldfish, and T is the event a customer buys a tortoise. It is known that $P(G) = 0.3$, $P(T) = 0.4$, and the probability that a customer buys neither a goldfish nor a tortoise is 0.42.

(a) Determine whether the events G and T are independent, justifying your answer. **[3 marks]**

(b) Find the probability that a customer buys a goldfish, given that they buy a tortoise. **[3 marks]**

Question 8 (5 Marks) — Paper 1 (No Calculator Allowed)

A mobile phone retailer receives stock from three different suppliers: A, B, and C. Supplier A provides 50% of the stock, Supplier B provides 30%, and Supplier C provides 20%. The probabilities that a phone is faulty from Suppliers A, B, and C are 1%, 3%, and 4% respectively. A customer buys a phone and finds it is faulty. Find the exact probability that it was supplied by Supplier B.

Question 9 (6 Marks) — Paper 2 (Calculator Allowed)

A rare disease affects 1% of a population. A medical test for the disease has a sensitivity of 95% (it correctly identifies 95% of those who have the disease) and a false positive rate of 5% (it incorrectly returns a positive result for 5% of healthy individuals). Let D represent having the disease, and $+$ represent testing positive.

(a) Draw a fully labeled Venn diagram representing this population, finding the probabilities of all four regions ($D \cap +$, $D \cap -$, $D' \cap +$, $D' \cap -$). **[4 marks]**

(b) Hence, find the probability that a person actually has the disease, given they tested positive. **[2 marks]**

Question 10 (4 Marks) — Paper 1 (No Calculator Allowed)

Urn A contains 3 red balls and 2 blue balls. Urn B contains 4 red balls and 1 blue ball. A fair coin is flipped. If the coin lands on Heads, a ball is drawn at random from Urn A. If the coin lands on Tails, a ball is drawn at random from Urn B. A ball is drawn and it is red. Find the exact probability that the coin landed on Heads.

SECTION C: HARD / VERY HARD (Synthesis & Proof)

Question 11 (6 Marks) — Paper 2 (Calculator Allowed)

At a second pet shop, let the events G and T represent buying a goldfish and a tortoise respectively. The probability of buying a tortoise is twice the probability of buying a goldfish: $P(T) = 2P(G)$. It is known that G and T are **independent events**, and the probability of buying neither is 0.28. Find $P(G|T \cup G)$, the probability a customer bought a goldfish given that they bought at least one of the two pets.

Question 12 (9 Marks) — Paper 2 (Calculator Allowed)

Events A, B, and C are represented in a universal set. It is given that A and B are independent events, where $P(A) = 0.25$ and $P(B) = 0.52$. Events B and C are mutually exclusive. You are also given that $P(A \cap C) = 0.09$, and $P(A \cup B \cup C) = 0.95$.

(a) Find $P(A \cap B)$. **[1 mark]**

(b) Using the formula for the union of three sets, find $P(C)$. **[4 marks]**

(c) Find $P(A' \cap B')$. **[2 marks]**

(d) Find $P(A|C')$. **[2 marks]**

Question 13 (5 Marks) — Paper 2 (Calculator Allowed)

[The Monty Hall Problem]
A game show features three closed doors. Behind one door is a car, and behind the other two are goats. The car is equally likely to be behind any door. You select Door 1. The host, who knows where the car is, must open a different door revealing a goat. If the car is behind Door 1, the host opens Door 2 or Door 3 with equal probability. Otherwise, he is forced to open the only remaining door with a goat. The host opens Door 3 to reveal a goat. Let C_1 be the event the car is behind Door 1, and H_3 be the event the host opens Door 3. Use Bayes' Theorem to calculate $P(C_2|H_3)$, the probability the car is behind Door 2.

Question 14 (5 Marks) — Paper 1 (No Calculator Allowed)

Events A and B are such that $P(A|B) = 0.5$, $P(A \cup B) = 0.9$, and $P(A') = 0.2$. Construct a Venn diagram to find the exact probability of $P(B|A)$.

Question 15 (5 Marks) — Paper 1 (No Calculator Allowed)

Two identical-looking bags contain coloured marbles. Bag 1 contains 4 White marbles and 3 Black marbles. Bag 2 contains 2 White marbles and 5 Black marbles. A bag is selected at random (each with probability $\frac{1}{2}$). Two marbles are then drawn from the selected bag *without replacement*. Both drawn marbles are found to be White. Calculate the exact probability that Bag 1 was chosen.