

IB MATHEMATICS AI HL

UNIT 2: FUNCTIONS

Composite, Inverse & Transformations

Instructions to Candidates

- This question booklet contains **15 questions**.
- The paper targets **AHL** syllabus components 2.7 and 2.8.
- Answer all questions, showing all step-by-step working clearly.

Difficulty Progression

- **Questions 1 - 5 (Easy):** Composite functions, simple inverses, and describing basic graph transformations visually.
- **Questions 6 - 10 (Medium):** Rational inverses, logarithmic composites, and domain restrictions for quadratics.
- **Questions 11 - 15 (Hard):** Self-inverse proofs, complex sequence transformations $f(a-bx)$, and finding inverses of quadratic exponentials using the quadratic formula.

SECTION A: EASY (Fundamentals)

Question 1 (4 Marks)

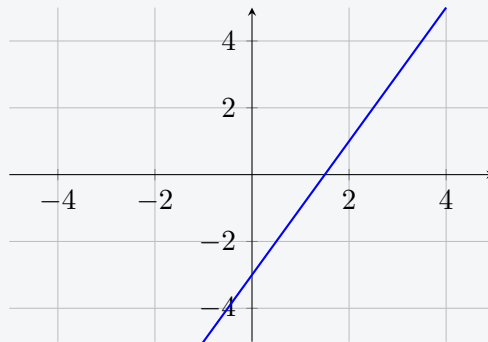
Let $f(x) = 2x - 5$ and $g(x) = x^2 + 1$.

(a) Find $(g \circ f)(x)$. [2 marks]

(b) Calculate $(f \circ g)(3)$. [2 marks]

Question 2 (4 Marks)

The graph of $y = f(x)$ is shown below as a solid blue line.



Sketch the inverse function $y = f^{-1}(x)$ on the same axes. State the geometrical relationship between the two graphs.

Question 3 (4 Marks)

The graph of $y = f(x)$ is transformed to $y = f(x - 4) + 2$. Describe the two geometric transformations fully.

Question 4 (4 Marks)

Let $h(x) = \frac{1}{x-2}$ for $x \neq 2$. State the domain and range of the inverse function $h^{-1}(x)$.

Question 5 (5 Marks)

The graph of $y = g(x)$ is stretched vertically by a scale factor of 3, then reflected in the x -axis. Write down the new function algebraically in terms of $g(x)$.

SECTION B: MEDIUM (Application & Modelling)

Question 6 (5 Marks)

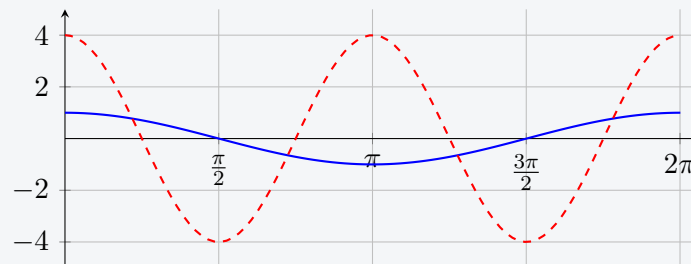
Find the inverse of the rational function $f(x) = \frac{2x-3}{x+1}$, where $x \neq -1$.

Question 7 (6 Marks)

Consider $f(x) = e^x$ and $g(x) = \ln(x+3)$ for $x > -3$. Find the composite function $(f \circ g)(x)$ and hence solve $(f \circ g)(x) = 10$.

Question 8 (6 Marks)

The graph below shows $y = \cos(x)$ for $0 \leq x \leq 2\pi$.



The dashed red curve is a transformation of the blue curve. Deduce the equation of the red curve and describe the sequence of geometric transformations.

Question 9 (6 Marks)

A quadratic function is given by $f(x) = x^2 - 6x + 8$ for $x \geq 3$. Find the algebraic expression for $f^{-1}(x)$.

Question 10 (6 Marks)

The point $A(2, 5)$ lies on $y = f(x)$. Find the exact coordinates of the corresponding point on the graph of $y = -f(x+1) - 3$.

SECTION C: HARD (Synthesis & Proof)**Question 11 (7 Marks)**

Show algebraically that the function $f(x) = \frac{3x+2}{x-3}$, for $x \neq 3$, is a self-inverse function.

Question 12 (8 Marks)

The graph of a function $y = g(x)$ is mapped onto the graph of $y = g(1 - 2x)$. Describe the precise sequence of geometric transformations required to achieve this, paying close attention to the order of operations inside the bracket.

Question 13 (8 Marks)

A piecewise function is given by:

$$f(x) = \begin{cases} \ln(x) & 0 < x < 1 \\ 2x - 2 & x \geq 1 \end{cases}$$

Find the inverse function $f^{-1}(x)$ and carefully state the domain restrictions for both pieces of the inverse function.

Question 14 (7 Marks)

Let $f(x) = x^3 + \ln(x) - 5$ for $x > 0$. The inverse function $f^{-1}(x)$ exists but cannot be found algebraically. Use your GDC to find the exact numerical value of $f^{-1}(12)$.

Question 15 (9 Marks)

Consider the function $h(x) = e^{2x} + 4e^x - 5$ for $x \geq 0$.

By treating this as a quadratic in terms of e^x , find the algebraic inverse function $h^{-1}(x)$. Justify why you must reject one of the roots from the quadratic formula.