

IB MATHEMATICS AA HL

AHL TOPIC 2 PRACTICE

Odd/Even Functions, Inverses, Inequalities & Modulus

Instructions to Candidates

- This practice paper contains **20** questions progressing from Easy to Very Hard.
- Each question indicates whether it is styled for **Paper 1 (No Calculator)** or **Paper 2 (Calculator Allowed)**.
- The paper tests syllabus topics AHL 2.14, 2.15, and 2.16: Odd and even functions, self-inverse functions, restricted domains for inverses, solving modulus equations and inequalities, and mapping $y = |f(x)|$ and $y = 1/f(x)$.
- Answer all questions, showing all your working clearly.
- Total marks available: **90**.

Difficulty Progression

- **SECTION A (Easy):** Identifying odd/even functions algebraically, basic domain/range swaps for inverse functions, simple modulus equations, and basic transformations.
- **SECTION B (Medium):** Proving logarithmic functions are odd/even, finding inverse functions of quadratics with restricted domains, solving modulus inequalities by squaring, and rational inequalities.
- **SECTION C (Hard):** Complex absolute value inequalities with fractions, finding intersection points of $f(x)$ and $f^{-1}(x)$, piecewise modulus equations, and transforming graphs to $y = 1/f(x)$.

SECTION A: EASY (Fundamentals)**Question 1 (3 Marks) — Paper 1 (No Calculator Allowed)**

Determine algebraically whether the function $f(x) = \frac{x^3}{x^2+1}$ is an odd function, an even function, or neither.

Question 2 (2 Marks) — Paper 1 (No Calculator Allowed)

A function $f(x)$ has a domain of $1 \leq x \leq 8$ and a range of $-5 \leq y \leq 10$. Given that the inverse function $f^{-1}(x)$ exists, state the domain and range of $f^{-1}(x)$.

Question 3 (3 Marks) — Paper 1 (No Calculator Allowed)

Solve the modulus equation $|2x - 5| = 9$ algebraically.

Question 4 (3 Marks) — Paper 1 (No Calculator Allowed)

Describe the sequence of three geometric transformations that maps the graph of $y = f(x)$ to the graph of $y = -f(2x) + 3$.

Question 4 (4 Marks) — Paper 1 (No Calculator Allowed)

The function f is defined as $f(x) = \frac{ax+b}{cx-a}$, where a, b , and c are non-zero constants and $x \neq \frac{a}{c}$. Show algebraically that $f(x)$ is a self-inverse function.

Question 6 (3 Marks) — Paper 2 (Calculator Allowed)

Use your graphic display calculator to solve the inequality $x^2 - 4x - 5 < 0$.

Question 7 (3 Marks) — Paper 1 (No Calculator Allowed)

Find the exact coordinates of the x -intercepts of the graph of $y = |2x - 6| - 4$.

SECTION B: MEDIUM (Application & Algebraic Methods)**Question 8 (4 Marks) — Paper 1 (No Calculator Allowed)**

Prove algebraically that the logarithmic function $g(x) = \ln\left(\frac{1+x}{1-x}\right)$ is an odd function for its entire domain $-1 < x < 1$.

Question 9 (4 Marks) — Paper 2 (Calculator Allowed)

A function is defined as $f(x) = 2(x+3)^2 - 5$ for $x \geq -3$. Find the expression for the inverse function $f^{-1}(x)$.

Question 10 (5 Marks) — Paper 2 (Calculator Allowed)

Solve the modulus inequality $|3x - 2| \geq |x + 4|$.

Question 11 (5 Marks) — Paper 1 (No Calculator Allowed)

Solve the rational inequality analytically:

$$\frac{x+5}{x-2} \leq 2$$

Question 12 (5 Marks) — Paper 1 (No Calculator Allowed)

Given the functions $f(x) = \frac{1}{x+2}$ for $x \neq -2$, and $g(x) = 3x - 1$. Find the explicit expression for the inverse composite function $(f \circ g)^{-1}(x)$.

Question 13 (6 Marks) — Paper 1 (No Calculator Allowed)

Let $f(x) = x^2 - 4x + 3$.

(a) On one set of axes, sketch the graph of $y = |f(x)|$. **[3 marks]**

(b) On a separate set of axes, sketch the graph of $y = f(|x|)$. **[3 marks]**

Ensure you clearly label the x -intercepts and y -intercept on both sketches.

Question 14 (5 Marks) — Paper 1 (No Calculator Allowed)

Solve the equation $|x^2 - 9| = 2x$ algebraically. Ensure you verify the validity of all potential roots.

SECTION C: HARD / VERY HARD (Synthesis & Proof)**Question 15 (6 Marks) — Paper 2 (Calculator Allowed)**

Solve the following rational modulus inequality:

$$\left| \frac{2x - 1}{x + 3} \right| > 1$$

Question 16 (5 Marks) — Paper 1 (No Calculator Allowed)

The quadratic function $f(x) = x^2 - 6x + 11$ is defined for $x \geq 3$. Find the exact coordinates of the point where the graph of $y = f(x)$ intersects the graph of its inverse function, $y = f^{-1}(x)$.

Question 17 (6 Marks) — Paper 1 (No Calculator Allowed)

[Odd/Even Decomposition]

Any function $f(x)$ can be expressed as the sum of an even function $E(x)$ and an odd function $O(x)$, such that $f(x) = E(x) + O(x)$.

- (a) Show algebraically that $E(x) = \frac{f(x) + f(-x)}{2}$. **[3 marks]**
- (b) Hence, find the explicit expressions for $E(x)$ and $O(x)$ when $f(x) = e^x$. **[3 marks]**

Question 18 (6 Marks) — Paper 1 (No Calculator Allowed)

Solve the equation $|x - 2| + |x + 3| = 7$ algebraically by considering the different piecewise regions of the x -axis.

Question 19 (5 Marks) — Paper 1 (No Calculator Allowed)

A continuous function $y = f(x)$ has a local maximum at $(-2, 4)$ and a horizontal asymptote of $y = 1$. The graph crosses the x -axis at $x = -4$ and $x = 0$. For the reciprocal graph $y = \frac{1}{f(x)}$, state:

- (a) The domain. [2 marks]
- (b) The coordinates and nature of the new turning point. [2 marks]
- (c) The equation of the horizontal asymptote. [1 mark]

Question 20 (7 Marks) — Paper 2 (Calculator Allowed)

Consider the nested modulus function $g(x) = ||x - 2| - 3|$. Use your graphic display calculator or algebraic methods to solve the equation:

$$g(x) = \frac{1}{2}x + 1$$

