IB MATHEMATICS AI HL
UNIT 2: FUNCTIONS

Scaling & Linearising Data

Instructions to Candidates

- This question booklet contains **15 questions**.
- The paper targets **AHL** syllabus component 2.10.
- Answer all questions, showing all step-by-step working clearly.

Difficulty Progression

- **Questions 1 - 5 (Easy):** Expanding log expressions, visualising basic semi-log and log-log graphs.
- **Questions 6 - 10 (Medium):** Extracting data from scatter diagrams, finding regression lines, extrapolation.
- **Questions 11 - 15 (Hard):** Non-standard linearisations (e.g. $y = ab^{x^2}$), algebraic proofs of power models, comparing Pearson (r) values to justify model choice.

SECTION A: EASY (Fundamentals)

Question 1 (4 Marks)

Use the laws of logarithms to expand $\ln\left(\frac{A}{B^3}\right)$ into a linear expression.

Question 2 (4 Marks)

Take the natural logarithm (ln) of both sides of the exponential model $y = ae^{kx}$, and show that it forms a straight line equation $Y = mX + C$. Identify Y, X, m and C.

Question 3 (4 Marks)

Take the base-10 logarithm ($\log_{10}$) of both sides of the power model $y = ax^n$, and show that it forms a straight line equation $Y = mX + C$.

Question 4 (4 Marks)

A semi-log graph is constructed. A line of best fit is formed, yielding $\ln y = 0.5x + 1.2$. By manipulating this equation, find the exact exponential model for y in terms of x.

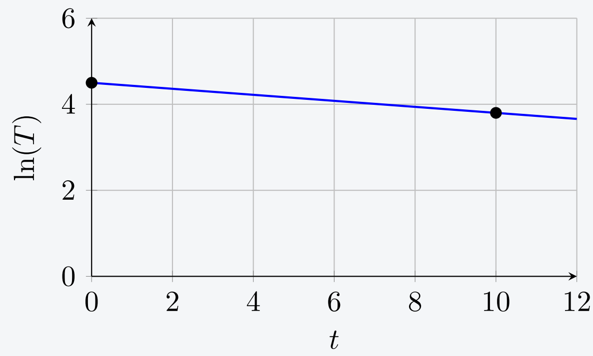
Question 5 (5 Marks)

A log-log graph plots $\log_{10} y$ against $\log_{10} x$. The line of best fit has a gradient of 3 and a Y-intercept of 2. State the power model $y = ax^n$.

SECTION B: MEDIUM (Application & Modelling)

Question 6 (5 Marks)

Data for a cooling cup of coffee is plotted on a semi-log graph of $\ln(T)$ vs time t.



The line passes through $(0, 4.5)$ and $(10, 3.8)$. Find the gradient and Y-intercept, and state the exponential cooling model.

Question 7 (6 Marks)

A planet's period P and distance D follow a power law $P = aD^n$. On a log-log graph ($\ln P$ vs $\ln D$), the points $(0, 1.4)$ and $(2, 4.4)$ are recorded. Find the exact values of a and n.

Question 8 (6 Marks)

Two raw data points for an exponential relationship $y = ab^x$ are $(2, 20)$ and $(5, 150)$. By setting up simultaneous equations, find a and b.

Question 9 (6 Marks)

A student linearises data by plotting $\log_{10} y$ against $\log_{10} x$ and finds the regression equation $Y = 2.4X - 0.5$. Find the value of y when $x = 100$.

Question 10 (6 Marks)

Explain why predicting a value far outside the given data range on a linearised semi-log graph is unreliable. State the mathematical term for this.

SECTION C: HARD (Synthesis & Proof)

Question 11 (7 Marks)

A biologist records bacteria growth. At $t = 1$, $N = 40$. At $t = 4$, $N = 1080$. Assume an exponential model $N = Ae^{kt}$. Solve algebraically for exact values of A and k.

Question 12 (8 Marks)

A dataset follows a non-standard relationship given by $y = ab^{x^2}$.
By taking the natural logarithm of both sides, show how this data can be linearised. State what should be plotted on the Y-axis and X-axis to produce a straight line, and define the gradient and Y-intercept of that line in terms of a and b.

Question 13 (8 Marks)

The equation of a linearised log-log graph is $\ln y = m \ln x + \ln c$. Prove algebraically that the original relationship must be a power model, and cannot be an exponential model.

Question 14 (7 Marks)

A student plots $\ln y$ against x and finds a Pearson's correlation coefficient of $r = 0.992$. They then plot $\ln y$ against $\ln x$ and find $r = 0.814$. Conclude which model (exponential or power) is a better fit, and justify your reasoning mathematically.

Question 15 (9 Marks)

Consider the relationship $y = \frac{50}{2x-1}$.
A student wants to linearise this data to find a straight line of best fit.
(a) What variables should the student plot on the vertical and horizontal axes to create a linear graph? [**4 marks**]
(b) What will the gradient and intercept of this new line be? [**5 marks**]