IB MATHEMATICS AI HL
UNIT 3: GEOMETRY

Vectors & Kinematics

Instructions to Candidates

- This question booklet contains **15 questions**.
- The paper targets **AHL** syllabus components 3.10, 3.11, and 3.12.
- Answer all questions, showing all step-by-step working clearly.

Difficulty Progression

- **Questions 1 - 5 (Easy):** Magnitude, unit vectors, position vectors, and basic line equations.
- **Questions 6 - 10 (Medium):** Intersection of lines, relative position vectors, distances, and dot products.
- **Questions 11 - 15 (Hard):** Minimum distance (closest approach), 3D skew lines, circular motion, and non-constant acceleration calculus.

SECTION A: EASY (Fundamentals)

CG50 Tip: Vector Arithmetic

Did you know your calculator can handle 3D vectors natively? In `MENU 1 (Run-Matrix)`, press `MATH (F4)` → `MAT/VCT (F1)` → `2x1` or `3x1` to type vectors directly on screen to add, subtract, or multiply them by scalars!

Question 1 (4 Marks)

Consider the vector $v = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}$.

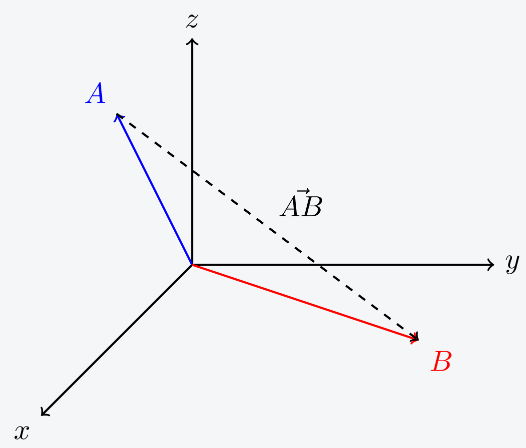
(a) Calculate the exact magnitude of v. [**2 marks**]
(b) Find the unit vector in the direction of v. [**2 marks**]

Question 2 (4 Marks)

A particle moves with a constant velocity of $v = 4i - 3j$ m/s. Its initial position at time $t = 0$ is given by the position vector $r_0 = -2i + 7j$ m.
Write down the vector equation of the particle's path and find its position vector at $t = 5$ seconds.

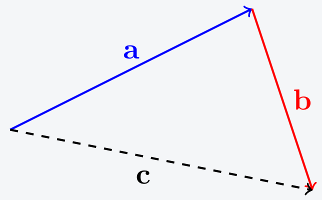
Question 3 (4 Marks)

The points $A(2, 4, -1)$ and $B(5, -2, 3)$ lie in a 3D coordinate space.



Find the vector equation of the line passing through A and B in the form $r = a + \lambda b$.

Question 4 (4 Marks)



Using the vector diagram above, express the dashed vector **c** in terms of **a** and **b**. Explain the geometric principle used.

Question 5 (5 Marks)

A particle moves in a straight line from point $P(-3, 5)$ to $Q(9, 1)$.
(a) Write down the vector equation of the line passing through P and Q. [**2 marks**]
(b) Convert your vector equation into the Cartesian form $y = mx + c$. [**3 marks**]

SECTION B: MEDIUM (Application & Modelling)

Question 6 (5 Marks)

Two submarines, Alpha and Beta, are tracked on a naval radar. Their positions at time t hours are:

$$r_A = \begin{pmatrix} -4 \\ 10 \end{pmatrix} + t\begin{pmatrix} 6 \\ -2 \end{pmatrix} \quad \text{and} \quad r_B = \begin{pmatrix} 14 \\ 2 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

Show algebraically that the paths of the submarines cross, but they do not collide.

Question 7 (6 Marks)

A drone is programmed to fly in a straight line. Its position vector r (in km) at time t (in hours) is $r = \begin{pmatrix} -5 \\ 12 \end{pmatrix} + t\begin{pmatrix} 8 \\ -6 \end{pmatrix}$.

Calculate the exact speed of the drone in km/h, and find its distance from the origin at $t = 2$.

Question 8 (6 Marks)

Ship P has position vector $r_P = (3 + 2t)i + (4 - t)j$. Ship Q has position vector $r_Q = (1 + t)i + (6 + 2t)j$.

(a) Find an expression for the relative position vector of Ship P from Ship Q, $\vec{QP}$. [**3 marks**]

(b) Calculate the exact distance between the two ships at $t = 3$ hours. [**3 marks**]

Question 9 (6 Marks)

Find the angle between the velocity vectors $u = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$.

Give your answer in degrees correct to 1 decimal place.

Question 10 (6 Marks)

A wind is blowing with a velocity vector $w = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$ km/h. A bird is flying with a velocity vector $v_b = \begin{pmatrix} 8 \\ -10 \end{pmatrix}$ km/h relative to the air.

Calculate the resultant true velocity vector of the bird relative to the ground, and find its true speed.

SECTION C: HARD (Synthesis & Proof)

Question 11 (7 Marks)

An asteroid is moving through space with position vector $r_A = \begin{pmatrix} 50 \\ 20 \\ -10 \end{pmatrix} + t\begin{pmatrix} -8 \\ 4 \\ 2 \end{pmatrix}$. A space station is located at $S(10, 30, -5)$.
Let $D(t)$ be the distance between the asteroid and the space station at time t.
(a) Find an algebraic expression for $D(t)^2$ in terms of t. [**3 marks**]
(b) By graphing $D(t)^2$ on your GDC or using calculus, find the exact time t when the asteroid is closest to the space station, and state this minimum distance. [**4 marks**]

Question 12 (8 Marks)

Consider the following two lines in 3D space:

$$L_1 : r = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \quad \text{and} \quad L_2 : r = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + \mu\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

Show algebraically that these two lines are skew (they are neither parallel nor do they intersect).

Question 13 (8 Marks)

A satellite orbits the Earth in circular motion. Its position vector at time t minutes is modelled by:

$$r(t) = 4000\cos\left(\frac{\pi}{60}t\right) i + 4000\sin\left(\frac{\pi}{60}t\right) j \text{ km}$$

(a) Find the velocity vector $v(t)$ by differentiating the position vector. [**3 marks**]
(b) Prove algebraically that the velocity vector is always perpendicular to the position vector for this satellite. [**5 marks**]

Question 14 (7 Marks)

An object is moving with a variable velocity vector given by $v(t) = \begin{pmatrix} 3t^2 \\ 4-2t \end{pmatrix}$ m/s.

Given that its initial position at $t = 0$ is $r_0 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ m, use integration to find the position vector of the object at $t = 4$ seconds.

Question 15 (9 Marks)

Two aircraft, Alpha and Beta, are flying in 3D space. Their coordinates at time t minutes are given by:

$$A(t) = (100 + 4t, 200 - 2t, 5000 + 10t) \quad \text{and} \quad B(t) = (150 - 3t, 180 + t, 5050 - 5t)$$

Calculate the exact time t when the vertical altitude difference between the two aircraft is exactly 100 meters, and determine the exact 3D distance between them at that moment.