Unit 1: Sequences and Series
IB Math AA SL

Answer all questions. Show all working where appropriate. Use your graphic display calculator (GDC) to efficiently solve or verify equations where possible.

1. **[Paper 1 Style, Short Answer, Easy, 6 marks]**
The second term, u_2, of a geometric sequence is 44 and the third term, u_3, is 55.

   (a) Find the common ratio, r, of the sequence.

   (b) Find the first term of the sequence, u_1.

   (c) Find S_5, the sum of the first 5 terms of the sequence.

2. **[Paper 1 Style, Short Answer, Easy, 3 marks]**
The sum of the first 16 terms of an arithmetic sequence is 920. Find the common difference, d, of the sequence if the first term is 27.5.

3. **[Paper 2 Style, Short Answer, Easy, 6 marks]**
Students are arranged for a graduation photograph in rows which follow an arithmetic sequence. There are 20 students in the fourth row and 44 in the 10th row.

   (a) Find the common difference, d, and the first term, u_1, of the sequence.

   (b) Given there are 20 rows of students in the photograph, calculate how many students there are altogether.

4. **[Paper 1 Style, Short Answer, Easy, 6 marks]**
A geometric sequence has $u_1 = 135$ and $u_4 = 5$.

   (a) Find the common ratio, r.

   (b) Find u_3 and u_7, giving your answers as exact fractions.

   (c) Find S_∞.

5. **[Paper 1 Style, Short Answer, Medium, 3 marks]**
A sequence can be defined by $a_n = 32 - 7n$, for $n \in \mathbb{Z}^+$. Write an expression for the sum of the first 12 terms using sigma notation (Σ) and calculate its exact value.

6. **[Paper 2 Style, Short Answer, Medium, 5 marks]**
An arithmetic sequence with a common difference of -3.5 has a first term of 77.

   (a) Given that the kth term of the sequence is zero, find the value of k.

   (b) Find the maximum value of the sum of the first n terms of the sequence.

7. **[Paper 2 Style, Short Answer, Medium, 6 marks]**
The sixth term of an arithmetic sequence is equal to 3 and the sum of the first 12 terms is 12. Find the common difference and the first term.

8. **[Paper 2 Style, Short Answer, Medium, 8 marks]**
Marie is an athlete returning to running after an injury. She decides to run 4 km the first week and increase this by 1.5 km each week.

   (a) Find the distance Marie runs in the 10th week.

   (b) Find the week in which Marie runs exactly 26.5 km.

   (c) Marie's coach says she can start preparing for her next race once she has run a total of 220 km. Find the week in which Marie will complete this.

9. **[Paper 1 Style, Short Answer, Medium, 6 marks]**
In a geometric sequence, the third term is 160 and the common ratio is $\frac{1}{4}$.

   (a) Find the first term, u_1.

   (b) Find u_6.

   (c) Find the exact value of the infinite sum of the sequence.

10. **[Paper 2 Style, Longer Question, Hard, 8 marks]**
The 18th term of an arithmetic sequence is 54 and the common difference is 2.2.

    (a) Find S_{18}, the sum of the first 18 terms.

    (b) The first and 18th terms of the arithmetic sequence are the first and second terms respectively of a geometric sequence. Find the smallest value of n such that $S_n > 10000$ for the geometric sequence.

11. [**Paper 2 Style, Longer Question, Hard, 10 marks**]
The fifth term of a geometric sequence is 1 and the common ratio, r, is $\frac{1}{3}$.

(a) Find S_5, the sum of the first five terms.

(b) Find the exact value of the infinite sum.

(c) The first and fifth terms of the geometric sequence are the 20th and 10th terms respectively of an arithmetic sequence. Find the largest integer value of n such that $S_n < 1000$ for the arithmetic sequence.

12. [**Paper 2 Style, Longer Question, Hard, 8 marks**]
The third term of a geometric sequence is 270 and the sixth term is -911.25.

(a) Find the 10th term of the sequence.

(b) Find the sum of the first 21 terms of the sequence, giving your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

13. [**Paper 1 Style, Longer Question, Hard, 8 marks**]
In an arithmetic sequence $u_1 = \log_b\left(\frac{x}{y}\right)$ and $u_2 = \log_b(x)$, where $k > 1$ and $x, y > 0$.

(a) Show that $d = \log_b(y)$.

(b) Let $x = b^5$ and $y = b^7$. Find the exact value of $\sum_{n=1}^{6} u_n$.

14. [**Paper 2 Style, Longer Question, Very Hard, 8 marks**]
The first term of both an arithmetic and a geometric sequence is 1 and both sequences have the same positive second term. The 20th term of the arithmetic sequence is five times the third term of the geometric sequence.

(a) Find the possible values of the second term.

(b) Find the corresponding values of the 10th term for each sequence.

15. [**Paper 1 Style, Extended Question, Very Hard, 15 marks**]
The first two terms of an infinite geometric sequence are $u_1 = 12$ and $u_2 = 9\cos^2\theta$, where $0 < \theta < 2\pi$, and $\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}$.

(a) Find an expression for r in terms of θ and find the range of possible values for r.

(b) Show that the sum of the infinite sequence can be expressed as $\frac{96}{5-3\cos(2\theta)}$.

(c) Find the exact values of θ which minimise the sum of the infinite sequence.