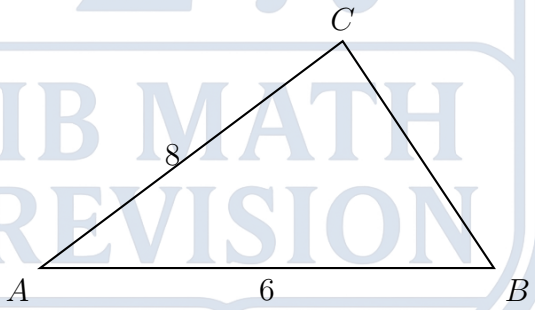
Unit 3: Non-Right-Angled Trigonometry
IB Math AA SL

Answer all questions. Show all working where appropriate. For Paper 1 questions, you must use analytical algebraic methods and give exact answers. For Paper 2 questions, use your graphic display calculator (GDC) efficiently and round to 3 significant figures.

1. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
The following diagram shows triangle ABC, with $AB = 6$ cm and $AC = 8$ cm.



Given that $\cos \hat{A} = \frac{5}{6}$:

(a) Find the exact value of $\sin \hat{A}$.

(b) Find the exact area of triangle ABC.

2. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
Owen, Henry and Tom are rugby players passing a ball in a park. Owen is at point O, Henry is at point H and Tom is at point T. The distance between Owen and Henry is 25 m and the distance between Henry and Tom is 18 m. The angle $O\hat{H}T$ is 96°.

(a) Find the distance of the pass from Owen directly to Tom, OT.

(b) Find the size of the angle $O\hat{T}H$.

3. **[Paper 1 Style, Non-Calculator, Easy, 5 marks]**
The following diagram shows triangle ABC, with $AB = 5$ and $BC = 4$. Given that $\sin \hat{B} = \frac{3}{5}$ and that angle $\hat{B}$ is obtuse:

   (a) Find the precise exact value of $\cos \hat{B}$.

   (b) Find the exact length of AC.

4. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
The diagram below shows a triangular field on a farm. $AB = 17$ m, $AC = 45$ m and angle $B\hat{A}C = 38°$.

   (a) The field is going to be used for livestock, so a fence is to be installed around its perimeter. Calculate the total length of fencing required.

   (b) Calculate the total area of the field.

5. **[Paper 1 Style, Non-Calculator, Medium, 6 marks]**
The following triangle shows triangle ABC, with $AB = 15$, $AC = 20$, and $BC = x$. Given that $\cos B\hat{A}C = \frac{2}{3}$:

   (a) Find the exact value of $\sin B\hat{A}C$.

   (b) Find the exact area of triangle ABC.

   (c) By finding the exact value of x, show that triangle ABC is an isosceles triangle.

6. **[Paper 2 Style, Calculator Required, Medium, 4 marks]**
The quadrilateral $ABCD$ represents a farm paddock, where $AB = 246$ m, $BC = 312$ m and $AD = 257$ m. Angle $D\hat{A}B = 96°$ and angle $B\hat{C}D = 78°$. A straight fence is built connecting points B and D to split the paddock into two distinct triangular regions. Calculate the length of the fence BD.

7. **[Paper 1 Style, Non-Calculator, Medium, 6 marks]**
The following diagram shows triangle ABC, with $AB = 3a$, $BC = a$ and $AC = 7$. Given that $\cos A\hat{B}C = \frac{1}{2}$:

   (a) Find the exact value of a.

   (b) Find the exact area of the triangle, giving your answer in the form $\frac{p\sqrt{3}}{r}$, where $p, r \in \mathbb{Z}^+$.

8. **[Paper 2 Style, Calculator Required, Medium, 6 marks]**
Adah would like to estimate the height of a tree located at point P on the edge of a riverbank, with the top of the tree at point Q. Due to a raging river, she is unable to reach the base of the tree. From point M she measures an angle of elevation of $20°$ to the top of the tree, and then from point N (which is on the edge of Adah's bank of the river) she measures an angle of elevation of $35°$ to the top of the tree. Between the points M and N she measures a horizontal distance of 12 m. Points M, N and P all lie on a single horizontal line, and point Q is vertically above point P.

   (a) Using the Sine Rule in triangle MNQ, calculate the length of NQ.

   (b) Calculate the vertical height of the tree, PQ.

9. **[Paper 1 Style, Non-Calculator, Hard, 5 marks]**
Consider an obtuse angle θ such that $\cos\theta = -\frac{2}{3}$. The following diagram shows triangle ABC, with $\hat{B} = 180° - \theta$, $\hat{A} = 2\theta - 180°$, $BC = a$ and $AC = b$. Show analytically using the Sine Rule that the ratio of the sides $\frac{b}{a} = \frac{3}{4}$. *(Hint: use double angle identities).*

10. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
The diagram below shows a police helicopter using a high beam light at point B to search an area on the ground between A and C. The length of the edge of the light beam that is furthest from the helicopter, BC, is 22 m and the angle of depression from the helicopter to point C is $47°$. Given that the area of the cross section of the search beam, ABC, is exactly 23 m^2:

   (a) Calculate the horizontal length of ground, AC, that is lit by the beam.

   (b) Find the size of the angle of the beam $A\hat{B}C$.

11. **[Paper 1 Style, Non-Calculator, Hard, 6 marks]**
Triangle ABC is such that the length of side AB is $3\sqrt{2}$ cm, the length of side BC is 3 cm, and the angle $B\hat{A}C = 30°$.

   (a) Use the sine rule to find the two possible exact values for the angle $A\hat{C}B$.

   (b) Explain geometrically why two valid triangles exist for these given measurements (the ambiguous case).

12. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
A small airline operates between three locations A, B and C. B is located 530 km from A on a bearing of 248°. C is located 300 km due East from the midpoint, M, of the line segment AB.

    (a) Find the size of the angle $A\hat{M}C$.

    (b) Calculate the straight-line distance AC.

13. **[Paper 1 Style, Non-Calculator, Very Hard, 7 marks]**
The following diagram shows triangle ABC, with $AC = \sqrt{41}$. Given that $\tan A\hat{B}C = -\frac{12}{5}$ and that the ratio of the length of side AB to the length of side BC is $10 : 13$:

    (a) Find the exact values of $\sin \hat{B}$ and $\cos \hat{B}$.

    (b) By defining $AB = 10x$ and $BC = 13x$, use the Cosine Rule to find the exact value of x.

    (c) Hence find the exact area of triangle ABC.

14. **[Paper 2 Style, Calculator Required, Very Hard, 7 marks]**
A 38 m high vertical cliff is perpendicular to the sea and the angle of depression from the top of the cliff (T) to a boat at sea (B) is 24°. Climbing the cliff is a rock climber (C) and the angle of elevation from the boat (B) to the climber (C) is 14°.

    (a) Draw a well-labelled diagram of this scenario.

    (b) Find the horizontal distance from the boat to the foot of the cliff (F).

    (c) Find how far the climber must still climb to reach the top of the cliff from their current position.

15. **[Paper 1 Style, Non-Calculator, Very Hard, 8 marks]**
Consider an acute angle θ such that $\cos\theta = \frac{2}{3}$. The following diagram shows triangle ABC, with $\hat{B} = \theta$, $\hat{A} = 2\theta$, $BC = a$ and $AC = b$. The line segment $[BA]$ is extended past A to a point D to form an isosceles triangle DAC, with $\hat{D} = \theta$.

    (a) Use the exterior angle theorem to show that $\angle ACD = \theta$, and hence deduce the lengths of the sides of triangle DAC in terms of b.

    (b) Find the exact value of $\sin C\hat{A}D$.

    (c) Show that the exact area of triangle DAC can be expressed as $\frac{\sqrt{5}}{8}a^2$.