

IB MATHEMATICS AA HL AHL TOPIC 1 PRACTICE

Counting Principles & Advanced Binomial Expansion

Instructions to Candidates

- This practice paper contains **15** questions progressing from Easy to Very Hard.
- Each question indicates whether it is styled for **Paper 1 (No Calculator)** or **Paper 2 (Calculator Allowed)**.
- The paper tests standard permutations/combinations and the **Higher Level extension** of the Binomial Theorem to fractional and negative indices.
- Answer all questions, showing all your working clearly.
- Total marks available: **62**.

Difficulty Progression

- **SECTION A (Easy):** Basic factorials, arrangements, standard $\binom{n}{r}$ evaluations, and straightforward positive integer binomial expansions.
- **SECTION B (Medium):** Intermediate counting with restrictions, algebraic binomial coefficients, and initial applications of $(1+x)^n$ for $n \in \mathbb{Q}$.
- **SECTION C (Hard):** Multi-bracket expansions, binomial approximations to decimal places, algebraic proof of identities, and synthesizing partial fractions with infinite binomial series.

SECTION A: EASY (Fundamentals)**Question 1 (2 Marks) — Paper 1 (No Calculator Allowed)**

Evaluate $\binom{7}{3}$ without a calculator and state exactly what this value represents in the context of combinatorics.

Question 2 (2 Marks) — Paper 1 (No Calculator Allowed)

Find the number of different ways to arrange all the letters of the word **NUMBER** in a straight line.

Question 3 (3 Marks) — Paper 2 (Calculator Allowed)

Find the coefficient of the term in x^3 in the binomial expansion of $(2 - x)^8$.

Question 4 (3 Marks) — Paper 1 (No Calculator Allowed)

Find the first three terms, in ascending powers of x , in the expansion of $(3 + x)^4$.

Question 5 (2 Marks) — Paper 1 (No Calculator Allowed)

A restaurant menu offers 5 starters, 8 main courses, and 4 desserts. Find the total number of different three-course meals a customer can choose, assuming they pick exactly one of each.

SECTION B: MEDIUM (Application)**Question 6 (3 Marks) — Paper 1 (No Calculator Allowed)**

A committee of 4 people is to be chosen from a group consisting of 6 men and 5 women. Find the number of different committees that can be formed if the committee must contain exactly 2 men and 2 women.

Question 7 (7 Marks) — Paper 1 (No Calculator Allowed)

Consider the binomial expansion of $\left(\frac{ax}{2} + \frac{3}{x^2}\right)^5$.

- (a) Find an expression, in terms of a , for the coefficient of the x^{-1} term. [5 marks]
- (b) Given that the coefficient of the x^{-1} term is exactly 90, find the value of the constant a . [2 marks]

Question 8 (4 Marks) — Paper 1 (No Calculator Allowed)

The coefficient of x^7 in the expansion of $x^3(ax + 3)^5$ is 1215. Find the possible values of the real constant a .

Question 9 (4 Marks) — Paper 1 (No Calculator Allowed)

[AHL Extension] Find the first three terms, in ascending powers of x , in the binomial expansion of $(1 + 2x)^{-3}$.

Question 10 (5 Marks) — Paper 1 (No Calculator Allowed)

[AHL Extension]

- (a) Find the binomial expansion of $\sqrt{1-x}$ up to and including the term in x^3 . [4 marks]
- (b) State the range of values of x for which this infinite expansion is mathematically valid. [1 mark]

SECTION C: HARD / VERY HARD (Synthesis & Proof)**Question 11 (4 Marks) — Paper 2 (Calculator Allowed)**

Find the coefficient of the x^{16} term in the expansion of $(2x^2 - x^3)^7$.

Question 12 (6 Marks) — Paper 2 (Calculator Allowed)

Consider the expansion of $(1 - 3x)^4(1 - 2kx)^2$, where k is a constant. Given that the coefficient of the x^6 term is 36, find the possible values of k .

Question 13 (4 Marks) — Paper 1 (No Calculator Allowed)

By using the factorial definition of $\binom{n}{r}$, prove algebraically that for $n > k > 0$:

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

Question 14 (8 Marks) — Paper 1 (No Calculator Allowed)

[AHL Extension - Partial Fractions & Series]

- (a) Express the rational function $\frac{4}{(1+x)(1-x)^2}$ in partial fractions. **[4 marks]**
- (b) Hence, find the Maclaurin/binomial expansion of $\frac{4}{(1+x)(1-x)^2}$ in ascending powers of x up to and including the term in x^2 . **[4 marks]**

Question 15 (5 Marks) — Paper 1 (No Calculator Allowed)

[AHL Extension]

- (a) Find the binomial expansion of $(1+3x)^{\frac{1}{2}}$ up to and including the term in x^2 . **[2 marks]**
- (b) By choosing a suitable value for x , use your expansion from part (a) to find an estimate for $\sqrt{1.03}$. Give your final answer correct to 5 decimal places. **[3 marks]**

REVISION