

IB MATHEMATICS AI HL

UNIT 4: STATISTICS

Confidence Intervals & Hypothesis Testing for Means

Instructions to Candidates

- This question booklet contains **15 questions**.
- The paper targets **AHL** syllabus components 4.16, 4.17, and 4.18.
- Answer all questions, showing all step-by-step working clearly.

Difficulty Progression

- **Questions 1 - 5 (Easy):** Defining H_0 and H_1 , p -value interpretations, and basic z -intervals.
- **Questions 6 - 10 (Medium):** 1-sample t -tests, paired t -tests, pooled 2-sample tests, and t -intervals from raw data.
- **Questions 11 - 15 (Hard):** Working backwards from Confidence Interval bounds, finding sample sizes, and algebraic critical regions.

SECTION A: EASY (Fundamentals)

Question 1 (4 Marks)

A school claims that its students study an average of 15 hours per week. A skeptical researcher believes the true average is actually less than 15 hours. They take a sample of 40 students to test this.

State the null hypothesis (H_0) and the alternative hypothesis (H_1) for this test.

CG50 Tip: Confidence Intervals

To quickly generate a Confidence Interval from summary statistics on the CG50, go to MENU 2 (STAT) → INTR (F4). Choose Z if the true population standard deviation (σ) is known, or t if it is unknown. Then choose 1-Sample and enter your stats!

Question 2 (4 Marks)

A machine fills bags with flour. The true population standard deviation of the fill weight is known to be $\sigma = 4.2$ grams. A random sample of 35 bags is taken, and the sample mean weight is $\bar{x} = 502.5$ grams.

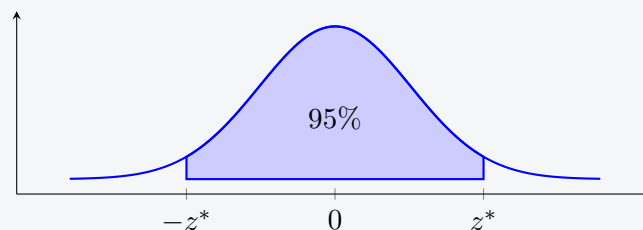
Calculate a 95% confidence interval for the true mean weight of the flour bags. Give your limits to 1 decimal place.

Question 3 (4 Marks)

A researcher conducts a hypothesis test at a 5% level of significance. The calculator outputs a p -value of 0.0382.

State the conclusion of the test, and give a clear mathematical reason for your answer.

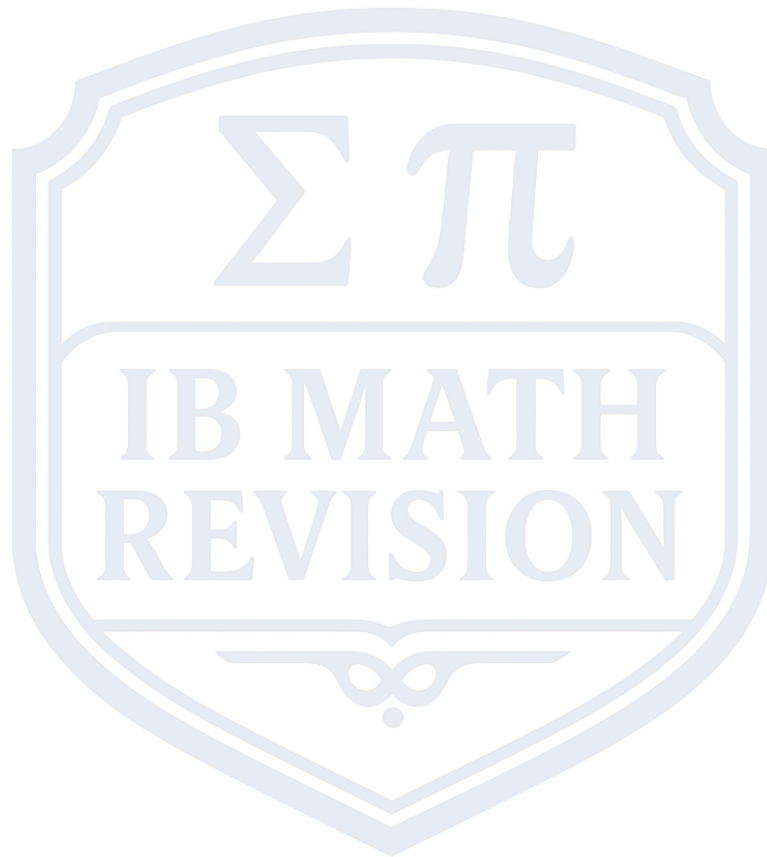
Question 4 (4 Marks)



The diagram above represents the standard normal distribution used to calculate a 95% confidence interval. Use your GDC to find the exact critical value z^* (correct to 3 decimal places) that bounds this central 95% region.

Question 5 (5 Marks)

A botanist is studying the height of a specific species of fern. They measure 12 ferns and calculate the sample mean to be 45.2 cm with a sample standard deviation of $s_{n-1} = 3.8$ cm. State the number of degrees of freedom for this sample, and calculate a 90% confidence interval for the population mean height.



SECTION B: MEDIUM (Application & Modelling)

Question 6 (5 Marks)

A factory produces lightbulbs that are supposed to last for an average of 1000 hours. A quality control manager tests a random sample of 8 bulbs and records their lifespans (in hours):

980, 1010, 995, 975, 985, 1005, 990, 970

Assuming the lifespans are normally distributed, perform a 1-sample t -test at the 10% significance level to determine if the true mean lifespan is **less than** 1000 hours. State your p -value and conclusion.

CG50 Tip: The Paired t -Test

If you have "Before" and "After" data for the exact same subjects, you must use a Paired t -Test. Enter the "Before" data in List 1, and the "After" data in List 2. Go to TEST \rightarrow $t \rightarrow$ 1-Sample. However, change the Data setting to List and make your list List 3, where List 3 is calculated in the spreadsheet as List 2 - List 1. Testing the difference against $\mu = 0$ is the Paired Test!

Question 7 (6 Marks)

Six athletes undertake a new training program to reduce their 100m sprint times. Their times (in seconds) before and after the program are recorded:

Athlete	A	B	C	D	E	F
Before	11.2	11.8	12.1	11.0	11.5	12.3
After	11.0	11.5	12.0	11.1	11.2	12.0

Perform a paired t -test at the 5% significance level to determine if the training program successfully reduced the sprint times. State your hypotheses, p -value, and conclusion.

Question 8 (6 Marks)

Two different brands of car tires, Brand X and Brand Y, are tested for braking distance (in metres) at 80 km/h. - Brand X (Sample of 15): Mean = 32.4, standard deviation $s_{n-1} = 2.1$ - Brand Y (Sample of 18): Mean = 34.1, standard deviation $s_{n-1} = 2.5$

Perform an **unpaired 2-sample t -test** at the 5% significance level to determine if there is a significant difference between the mean braking distances of the two brands.

Question 9 (6 Marks)

A 95% confidence interval for the mean weight of a population of cats is calculated to be (4.20 kg, 5.10 kg).

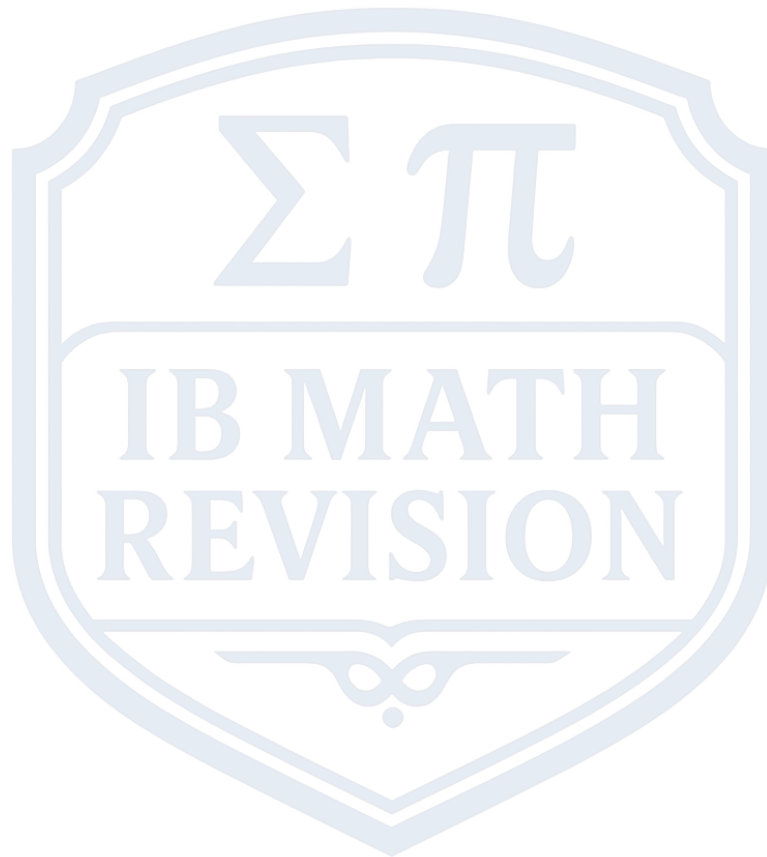
- (a) Write down the sample mean \bar{x} that was used to create this interval. [2 marks]
 (b) Write down the margin of error (E) for this confidence interval. [2 marks]

(c) Explain why it would be mathematically incorrect to say "There is a 95% probability that the true population mean lies between 4.20 and 5.10." [2 marks]

Question 10 (6 Marks)

A researcher wants to estimate the true mean height of adult men in a city. The population standard deviation is known to be $\sigma = 7.5$ cm.

Calculate the minimum sample size n required so that the researcher can be 99% confident that their sample mean is within 2 cm of the true population mean.



SECTION C: HARD (Synthesis & Proof)

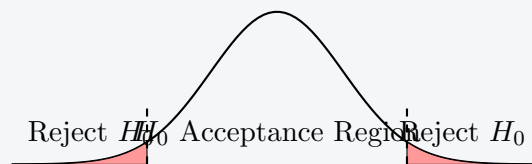
Question 11 (7 Marks)

A 90% confidence interval for a population mean μ was calculated using a sample of $n = 25$ observations. The population standard deviation σ was unknown, so the t -distribution was used. The calculated interval is: $120.4 < \mu < 129.6$.

Determine the exact sample standard deviation (s_{n-1}) that was calculated from the raw data to generate this interval.

Question 12 (8 Marks)

A two-tailed hypothesis test is conducted using a 1-sample Z -test at the 5% significance level. The null hypothesis is $H_0 : \mu = 50$. The population standard deviation is known to be $\sigma = 12$, and a sample of $n = 36$ is taken.



Determine the exact "critical regions" for the sample mean \bar{x} . In other words, find the boundaries for \bar{x} that will cause the researcher to reject the null hypothesis.

Question 13 (8 Marks)

Two groups of students take a memory test. Group 1 ($n_1 = 12$) has a sample mean of $\bar{x}_1 = 65.2$ and standard deviation $s_1 = 5.4$.

Group 2 ($n_2 = 14$) has a sample mean of $\bar{x}_2 = 70.8$ and standard deviation $s_2 = 6.1$.

A 2-sample t -test is run at the 5% significance level to test if $\mu_1 \neq \mu_2$.

(a) Perform the test assuming the population variances are **pooled** (equal variances). State the p -value. [4 marks]

(b) Perform the test assuming the population variances are **not pooled** (unequal variances). State the p -value. [4 marks]

Question 14 (7 Marks)

A farmer measures the daily milk yield (in litres) of 5 cows before and after changing their diet. Let $d = (\text{After} - \text{Before})$. The differences for the 5 cows are:

$$d_1 = 1.2, \quad d_2 = 0.8, \quad d_3 = x, \quad d_4 = 1.5, \quad d_5 = 0.5$$

The farmer runs a paired t -test and finds that the mean difference $\bar{d} = 1.1$.

(a) Find the missing value x . [2 marks]

(b) Calculate the p -value for the test $H_0 : \mu_d = 0$ vs $H_1 : \mu_d > 0$. [5 marks]

Question 15 (9 Marks)

A pharmaceutical company is testing a new blood pressure drug. They measure the blood pressure of a sample group, administer the drug for a month, and measure it again. The results of a paired t -test yield a p -value of 0.041.

(a) At the 5% significance level, state whether the company should accept or reject the null hypothesis. [2 marks]

(b) The company proudly publishes a paper claiming: "Our test proved with absolute certainty that the drug works on all humans." Based on IB Statistics syllabus concepts regarding Hypothesis Testing limitations, state two distinct reasons why this published claim is invalid. [4 marks]

(c) Calculate a 95% confidence interval for the mean difference if the sample size was $n = 20$, the mean difference was $\bar{d} = -5.4$, and the standard deviation of differences was $s_d = 10.2$. [3 marks]

