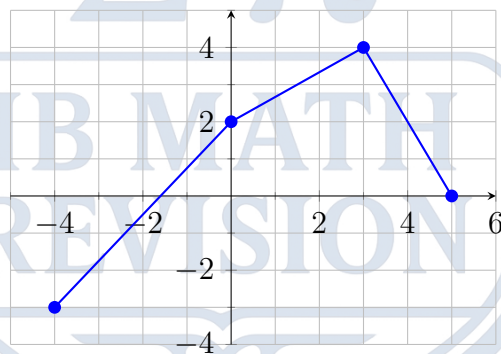


**Unit 2: Function Fundamentals (Graphical Focus)**  
**IB Math AA SL**

*Answer all questions. Show all working where appropriate. Many questions require you to sketch on the provided axes or draw your own. Use your GDC to verify shapes and intersections.*

1. [Paper 1 Style, Short Answer, Easy, 4 marks]

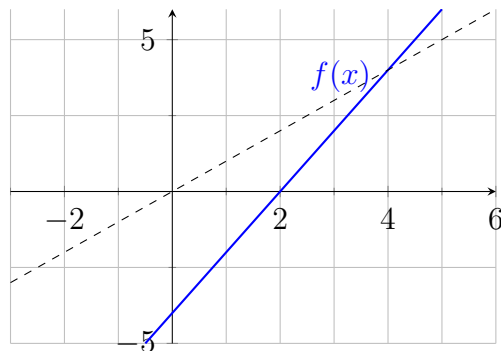
The diagram below shows the graph of a function  $y = f(x)$  for  $-4 \leq x \leq 5$ .



- Find the value of  $f(0)$ .
- Find the value of  $f^{-1}(4)$ .
- State the domain and range of  $f$ .

2. [Paper 1 Style, Short Answer, Easy, 4 marks]

The graph of the linear function  $f(x) = 2x - 4$  is shown below.

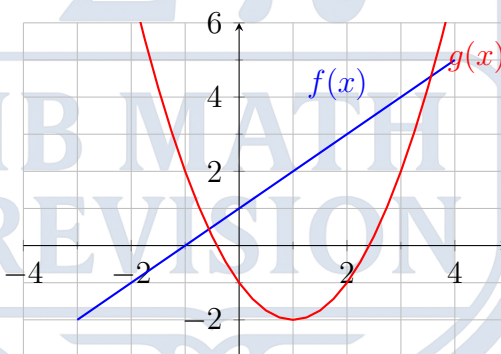


(a) Sketch the graph of the inverse function  $f^{-1}(x)$  on the same axes.

(b) Find the algebraic expression for  $f^{-1}(x)$ .

3. [Paper 1 Style, Short Answer, Easy, 4 marks]

The graphs of two functions,  $f$  and  $g$ , are shown on the same set of axes below.



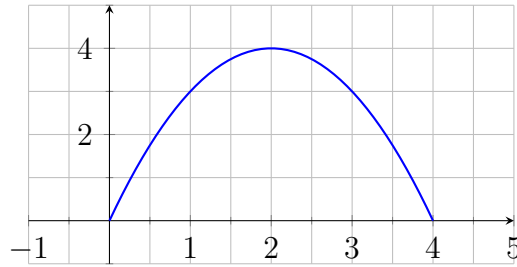
Find the value of:

(a)  $g(f(-1))$

(b)  $(f \circ g)(2)$

4. [Paper 1 Style, Short Answer, Easy, 5 marks]

Consider the function  $f(x) = -x^2 + 4x$ , restricted to the domain  $0 \leq x \leq 4$ .



- (a) Write down the range of  $f$ .
- (b) Using the graph, explain why the inverse function  $f^{-1}(x)$  does not exist for this domain.

5. [Paper 2 Style, Short Answer, Medium, 5 marks]

Let  $f(x) = 3^x - 2$ .

- (a) Sketch the graph of  $f(x)$  for  $-2 \leq x \leq 2$  on a set of axes. Clearly label the  $y$ -intercept and the horizontal asymptote.
- (b) Write down the domain and range of  $f^{-1}(x)$ .

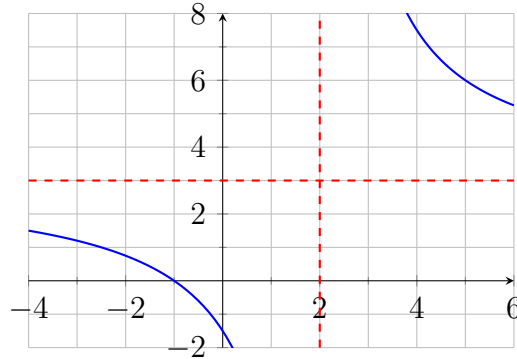
6. [Paper 2 Style, Short Answer, Medium, 5 marks]

Let  $g(x) = \ln(x + 1) + 2$ .

- (a) Sketch the graph of  $g(x)$  for  $-1 < x \leq 5$ . Clearly label the exact coordinates of the  $x$ -intercept and the vertical asymptote.
- (b) Find  $g^{-1}(x)$ .

7. [Paper 1 Style, Short Answer, Medium, 6 marks]

The rational function  $h(x) = \frac{ax+b}{x+c}$  is graphed below. The graph has a vertical asymptote at  $x = 2$  and a horizontal asymptote at  $y = 3$ . It crosses the  $x$ -axis at  $(-1, 0)$ .



- Deduce the values of  $a$ ,  $b$ , and  $c$ .
- Write down the equations of the asymptotes for  $h^{-1}(x)$ .

8. [Paper 1 Style, Short Answer, Medium, 5 marks]

A function is defined piecewise as:

$$f(x) = \begin{cases} x + 2 & \text{for } -4 \leq x < 0 \\ 2 - x^2 & \text{for } 0 \leq x \leq 2 \end{cases}$$

- Sketch the graph of  $y = f(x)$  on a set of axes.
- Find the exact range of  $f$ .
- Determine the value(s) of  $x$  such that  $f(x) = 0$ .

9. [Paper 2 Style, Short Answer, Medium, 5 marks]

Let  $f(x) = \frac{1}{2}x + 2$  and  $g(x) = x^2 - 4x + 3$ .

- Sketch the graph of  $(f \circ g)(x)$  using your GDC to find the vertex and  $y$ -intercept.
- Find the range of  $(f \circ g)(x)$ .

10. [Paper 2 Style, Longer Question, Medium, 6 marks]

The graph of  $f(x) = (x - 3)^2 - 1$  is restricted to a domain of  $x \geq 3$ .

- Sketch  $f(x)$  on this domain.
- Find the inverse function  $f^{-1}(x)$ .
- Sketch  $f^{-1}(x)$  on the same axes as  $f(x)$ , explicitly showing the line of reflection.

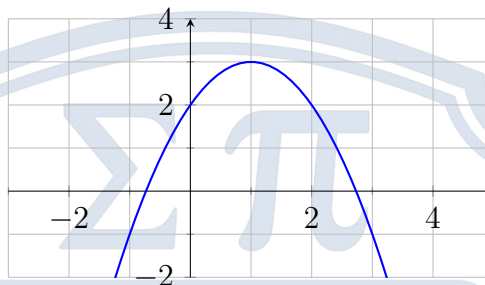
11. [Paper 1 Style, Longer Question, Hard, 7 marks]

Let  $f(x) = \sqrt{x+4}$  and  $g(x) = x - 2$ .

- Find the domain and range of  $f(x)$ .
- Sketch the graphs of  $f(x)$  and  $g(x)$  on the same grid.
- Use algebra to find the exact coordinates of the point(s) of intersection.

12. [Paper 2 Style, Longer Question, Hard, 6 marks]

The diagram below shows the graph of a function  $f(x)$ .



The function  $g$  is defined by  $g(x) = 2x - 1$ .

- From the graph, estimate the vertex of  $f(x)$  and hence write its equation in the form  $a(x - h)^2 + k$ .
- Sketch the graph of  $(g \circ f)(x)$  on a new set of axes, indicating the new vertex and  $y$ -intercept.

13. [Paper 1 Style, Short Answer, Hard, 6 marks]

Let  $f(x) = \frac{x+5}{x-1}$  for  $x \neq 1$ .

- Find  $f^{-1}(x)$ .
- Hence, or otherwise, what geometrical feature of the graph of  $f(x)$  explains why  $f(x) = f^{-1}(x)$ ? Sketch  $f(x)$  and the line  $y = x$  to support your answer.

14. [Paper 2 Style, Longer Question, Hard, 7 marks]

The graphs of  $f(x) = e^{0.5x}$  and its inverse  $f^{-1}(x)$  enclose a region with the coordinate axes and the line  $x = 4$ .

- Find the expression for  $f^{-1}(x)$ .
- Sketch both graphs on the same axes.
- Shade the region enclosed by  $y = f^{-1}(x)$ , the  $x$ -axis, and the line  $x = 4$ .

15. [Paper 1 Style, Short Answer, Hard, 6 marks]

Let  $f(x) = \sqrt{2x - 4}$  and  $g(x) = \frac{1}{x-3}$ .

- (a) State the domain of  $f$  and the domain of  $g$ .
- (b) Find the expression for  $(g \circ f)(x)$ .
- (c) Determine the maximum valid domain for  $(g \circ f)(x)$ .

16. [Paper 2 Style, Longer Question, Very Hard, 8 marks]

The graph of  $f(x)$  is a semi-circle given by  $f(x) = \sqrt{9 - x^2}$  for  $0 \leq x \leq 3$ .

- (a) Sketch the graph of  $f(x)$ .
- (b) Write down the domain and range of  $f$ .
- (c) Find the algebraic expression for  $f^{-1}(x)$ .
- (d) By adding  $f^{-1}(x)$  to your sketch, deduce the coordinates of the point of intersection between  $f$  and  $f^{-1}$ .

17. [Paper 1 Style, Extended Question, Very Hard, 7 marks]

Let  $f(x) = \ln x$  and  $g(x) = e^x$ .

- (a) Sketch the graphs of  $y = (f \circ g)(x)$  and  $y = (g \circ f)(x)$  on two separate grids.
- (b) Explain why the two graphs from part (a) are not identical, despite  $f$  and  $g$  being inverse functions. Refer to domains and ranges in your explanation.

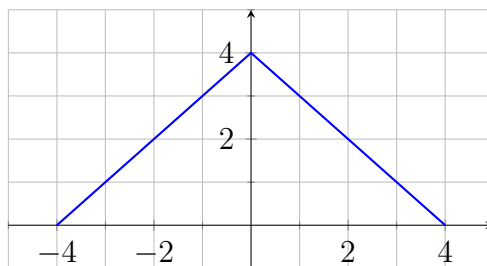
18. [Paper 2 Style, Extended Question, Very Hard, 8 marks]

Consider the rational function  $f(x) = \frac{4x-3}{2x-c}$ , where  $c$  is a constant. The graph of  $y = f(x)$  is symmetric about the line  $y = x$ .

- (a) Sketch a generic rational function that is symmetric over  $y = x$ . What must be true about its vertical and horizontal asymptotes?
- (b) Using your deduction from part (a), find the value of  $c$ .
- (c) Verify your answer by algebraically computing  $f^{-1}(x)$ .

19. [Paper 1 Style, Extended Question, Very Hard, 7 marks]

The graph of  $y = f(x)$  consists of two line segments connecting the points  $(-4, 0)$ ,  $(0, 4)$ , and  $(4, 0)$ .



- Write down the piecewise definition of  $f(x)$ .
- Let  $g(x) = (f \circ f)(x)$ . Using the graph or your piecewise definition, find the value of  $g(2)$  and  $g(-2)$ .
- Sketch the graph of  $y = g(x)$  for  $-4 \leq x \leq 4$ .

20. [Paper 2 Style, Extended Question, Very Hard, 8 marks]

Let  $f(x) = \frac{1}{2}x^2$  for  $x \geq 0$ . The graph of  $f$  and its inverse  $f^{-1}$  intersect at the origin  $O$  and another point  $P$ .

- Sketch  $f(x)$  and  $f^{-1}(x)$  on the same set of axes.
- Find the exact coordinates of  $P$ .
- Find the exact distance  $OP$ .
- A line is drawn perpendicular to  $y = x$  passing through the point  $(1, 1)$ . This line intersects  $f(x)$  at  $A$  and  $f^{-1}(x)$  at  $B$ . Sketch this line and find the distance  $AB$ .