Unit 5: Definite Integrals & Area
IB Math AA SL

Answer all 15 questions. Show all working. For Paper 1 questions, use analytical algebraic methods. For Paper 2 questions, use your graphic display calculator (GDC) efficiently.

1. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
   Find the exact value of the positive constant k such that:
   $$\int_0^k 3x^2\,dx = 64$$

2. **[Paper 2 Style, Calculator Required, Easy, 4 marks]**
   Consider the function $f(x) = \frac{1}{2}x^2 + 2$.
   (a) Using your graphic display calculator, calculate the exact value of the definite integral $\int_0^4 \left(\frac{1}{2}x^2 + 2\right) dx$.
   (b) A student estimates the area under the curve between $x = 0$ and $x = 4$ using a basic geometric method and gets an answer of 19. Calculate the percentage error of their approximation.

3. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
   Find the exact value of the following definite integral:
   $$\int_1^4 \frac{1}{x}\,dx$$

4. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
   The curve $y = -x^2 + 6x - 5$ forms an enclosed region with the x-axis.
   (a) Use your graphic display calculator to find the x-intercepts (roots) of the curve.
   (b) Write down the definite integral that represents the area of the enclosed region.
   (c) Evaluate the exact area of this region.

5. **[Paper 1 Style, Non-Calculator, Medium, 5 marks]**
The area enclosed by the curve $y = \frac{1}{x^2}$ (for $x > 0$), the x-axis, the vertical line $x = 1$, and the vertical line $x = a$ (where $a > 1$) is exactly 0.8 square units.

   (a) Find an expression for $\int_1^a \frac{1}{x^2}\,dx$ in terms of a.

   (b) Set up an equation and solve it analytically to find the exact value of a.

6. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
The graphs of $y = x^2$ and $y = x + 2$ enclose a finite region.

   (a) Use your graphic display calculator to find the x-coordinates of the points of intersection of the two graphs.

   (b) Write down the definite integral that represents the area of the enclosed region.

   (c) Calculate the exact area of this region.

7. **[Paper 1 Style, Non-Calculator, Medium, 6 marks]**
Consider a continuous function $h(x)$ such that:

$$\int_0^7 h(x)\,dx = 19 \quad \text{and} \quad \int_4^7 h(x)\,dx = 12$$

Find the exact values of:

   (a) $\int_0^4 h(x)\,dx$

   (b) $\int_4^7 \frac{4-h(x)}{5}\,dx$

8. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
The curve $y = x^3 - 4x$ crosses the x-axis at three points, creating two bounded regions between the curve and the x-axis.

   (a) Find the x-intercepts of the curve.

   (b) Explain mathematically why calculating the single integral $\int_{-2}^2 (x^3 - 4x)\,dx$ will **not** give the total area enclosed by the curve and the x-axis.

   (c) Use the absolute value function on your GDC to find the true total area of the two enclosed regions.

9. **[Paper 1 Style, Non-Calculator, Hard, 5 marks]**
Find the exact value of the following definite integral, giving your answer in the form $p \ln q$:

$$\int_1^5 \frac{3}{2x}\,dx$$

10. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
The diagram shows a sketch of part of the curves with equations:

$$y = x^2 - 3x + 4 \quad \text{and} \quad y = 4 - x^2 + 2x$$

(a) Find the x-coordinates of the two points where the graphs intersect.

(b) Show that the area of the region R bounded between the two curves is given by $\int_0^{2.5}(5x - 2x^2)\,dx$.

(c) Use calculus on your GDC to find the exact area of the shaded region R.

11. **[Paper 1 Style, Non-Calculator, Hard, 6 marks]**
The line $y = 6 - x$ and the curve $y = x^2$ intersect in the first quadrant.

(a) Find the exact x-coordinate of the point of intersection in the first quadrant.

(b) Analytically calculate the exact area of the region bounded by the y-axis, the curve $y = x^2$, and the line $y = 6 - x$.

12. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
The shaded region R is bounded by the two curves $y = 5x^2 - 12x + 8$ and $y = -3x^2 + 10x + 29$. The curves intersect at points A and B.

(a) By setting up and solving an appropriate quadratic equation, find the x-coordinates of points A and B.

(b) Find the area of region R, giving your answer as an exact value.

13. **[Paper 1 Style, Non-Calculator, Very Hard, 6 marks]**
Evaluate the following definite integral analytically. Give your answer in the form $\frac{\sqrt{a}}{b}$, where a and b are integers.

$$\int_0^{\frac{\pi}{6}} \cos\left(2x - \frac{\pi}{3}\right)\,dx$$

14. **[Paper 2 Style, Calculator Required, Very Hard, 7 marks]**
The shaded region R is bounded entirely by the x-axis, the line $y = 8x - 4$, and the cubic curve $y = -x^3 + x^2 + 10x + 8$. Using your graphic display calculator, or otherwise:

(a) Find the x-coordinate of the point of intersection between the curve and the line in the first quadrant.

(b) Find the x-coordinate where the line intersects the x-axis, and the x-coordinate where the curve intersects the positive x-axis.

(c) By splitting the region R into two separate integrals, show that the total area of region R is exactly $\frac{439}{12}$ square units.

15. **[Paper 1 Style, Non-Calculator, Very Hard, 6 marks]**
Let $f(x) = x^2 - x - 2$. By first determining the x-intercepts of the curve, calculate analytically the exact area of the region completely enclosed by the graph of $y = f(x)$ and the x-axis.