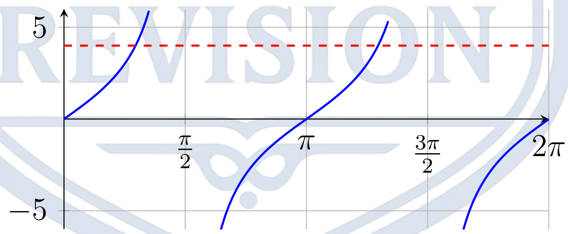
Unit 3: Solving Trigonometric Equations
IB Math AA SL

Answer all 15 questions. Show all working. For Paper 1 questions, use analytical algebraic methods and exact values. For Paper 2 questions, use your graphic display calculator (GDC) efficiently.

1. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
   Solve the equation $2\sin x - \sqrt{3} = 0$ for the interval $0 \le x \le 2\pi$. Give your answers exactly in radians.

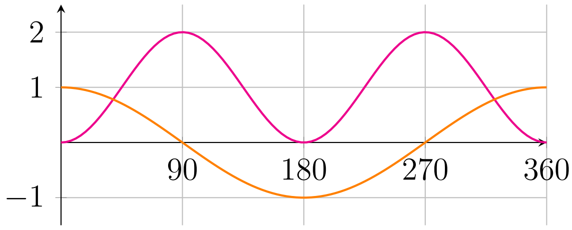
2. **[Paper 2 Style, Calculator Required, Easy, 4 marks]**
   The graphs of $y = 3\tan x$ and $y = 4$ are shown below for $0 \le x \le 2\pi$.



   Use your graphic display calculator to solve the equation $3\tan x = 4$ for $0 \le x \le 2\pi$. Give your answers correct to three significant figures.

3. **[Paper 1 Style, Non-Calculator, Easy, 5 marks]**
   Solve the equation $\sqrt{2}\cos(2x) + 1 = 0$ for the finite interval $0 \le x \le \pi$. Give your answers exactly in radians.

4. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
The diagram below shows the graphs of $f(x) = 2\sin^2 x$ and $g(x) = \cos x$ for $0° \leq x \leq 360°$.



(a) By equating $f(x)$ and $g(x)$, show that the points of intersection satisfy the quadratic trigonometric equation $2\cos^2 x + \cos x - 2 = 0$.

(b) Use the equation solver on your GDC to find the two possible values of $\cos x$.

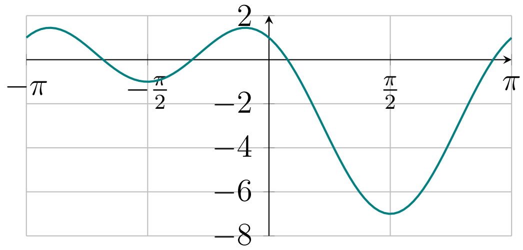
(c) Hence, find the two x-coordinates of the intersection points in degrees.

5. **[Paper 1 Style, Non-Calculator, Medium, 4 marks]**
Solve the equation $\tan^2 x - 3 = 0$ for $0 \leq x \leq 2\pi$. Give your answers exactly in radians.

6. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
A function is given by $f(x) = 3\sin\left(2x - \frac{\pi}{4}\right)$. Use your graphic display calculator to solve the equation $f(x) = 2$ in the interval $0 \leq x \leq \pi$. Give your answers to three significant figures.

7. **[Paper 1 Style, Non-Calculator, Medium, 6 marks]**
Solve the equation $2\sin^2 x + \cos x - 1 = 0$ for $0 \leq x \leq 2\pi$. *(Hint: use the Pythagorean identity to form a quadratic equation in* $\cos x$*).*

8. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
The function $y = 5\cos^2 x - 3\sin x - 4$ is plotted below over the domain $-\pi \leq x \leq \pi$.



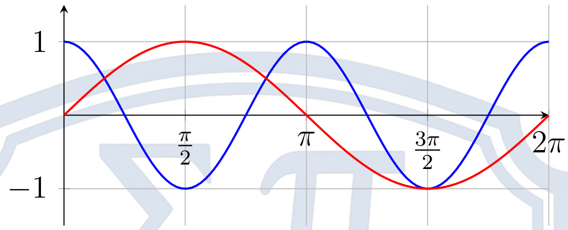
(a) Show algebraically that finding the roots of this function is equivalent to solving the equation $5\sin^2 x + 3\sin x - 1 = 0$.

(b) Use your graphic display calculator to find the exact x-coordinates of the four roots shown in the graph above.

9. **[Paper 1 Style, Non-Calculator, Hard, 6 marks]**
Solve the equation $\sin 2x = \sqrt{3}\cos x$ for $0 \le x \le 2\pi$. *(Hint: use the double angle identity for sine, then factorise).*

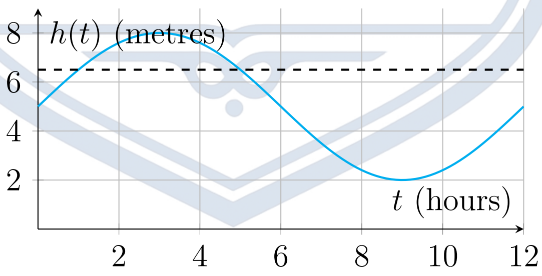
10. **[Paper 2 Style, Calculator Required, Hard, 5 marks]**
Solve the equation $\tan^2 x + 2\tan x - 8 = 0$ for $-\pi \le x \le \pi$. Give your answers to three significant figures.

11. **[Paper 1 Style, Non-Calculator, Hard, 6 marks]**
The graphs of $y = \cos 2x$ and $y = \sin x$ are shown below for $0 \le x \le 2\pi$. They intersect at three distinct points.



Use the double angle identity for cosine to solve the equation $\cos 2x = \sin x$ analytically, thereby finding the exact x-coordinates of the three intersection points.

12. **[Paper 2 Style, Calculator Required, Hard, 5 marks]**
The height of the tide in a harbour, h in metres, at time t hours after midnight is modelled by $h(t) = 3\sin\left(\frac{\pi}{6}t\right) + 5$.



A large cargo ship can only safely navigate the harbour when the tide height is strictly greater than 6.5 m (shown by the dashed line). Use your GDC to solve $h(t) = 6.5$ and find the total amount of time during the first 12 hours $(0 \le t \le 12)$ when the ship can safely navigate the harbour.

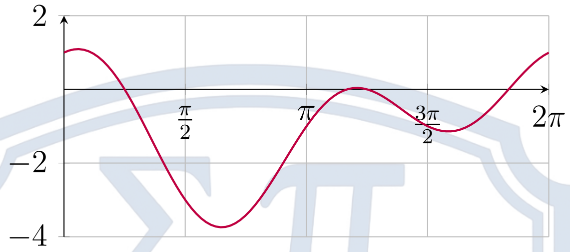
13. **[Paper 1 Style, Non-Calculator, Very Hard, 6 marks]**
Solve the equation $2\sin x\cos x = \cos^2 x - \sin^2 x$ for the interval $0 \le x \le \pi$.

14. **[Paper 2 Style, Calculator Required, Very Hard, 6 marks]**
Consider the equation $\frac{5\sin x - 2\cos x}{\cos x} = \tan^2 x$ for $0 \le x \le 2\pi$.

(a) By manipulating the left-hand side, show that the equation can be written as the quadratic $\tan^2 x - 5\tan x + 2 = 0$.

(b) Use your graphic display calculator to solve for all valid values of x in the given domain.

15. **[Paper 1 Style, Non-Calculator, Very Hard, 8 marks]**
Consider the function $f(x) = \sin 2x + \cos 2x - 1 + \cos x - \sin x$. The graph of $f(x)$ is plotted below for $0 \le x \le 2\pi$.



(a) Show analytically that $\sin 2x + \cos 2x - 1 = 2\sin x(\cos x - \sin x)$.

(b) Hence, factorise $f(x)$ and solve the equation $f(x) = 0$ to find the exact x-coordinates of the four roots shown in the graph for $0 < x < 2\pi$.