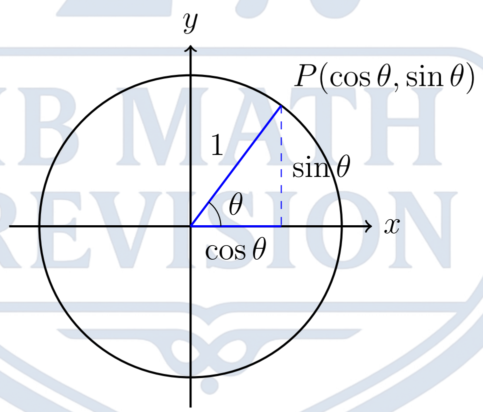
Unit 3: Trigonometric Identities (Graphical & Analytical)
IB Math AA SL

Answer all 10 questions. Show all working. For Paper 1 questions, use analytical algebraic methods. For Paper 2 questions, use your graphic display calculator (GDC) efficiently.

1. **[Paper 1 Style, Non-Calculator, Easy, 5 marks]**
   The diagram below shows a point $P(x, y)$ on the unit circle. A right-angled triangle is formed with the x-axis. The angle at the origin is θ.

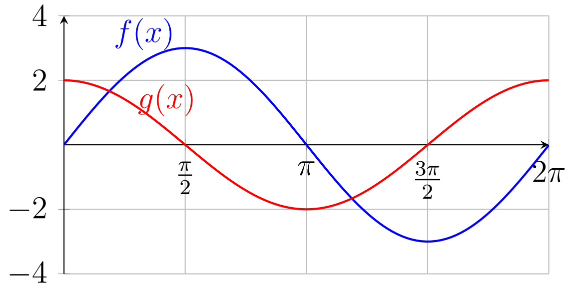


   (a) Using the geometric properties of the right-angled triangle shown, prove the Pythagorean identity: $\sin^2\theta + \cos^2\theta = 1$.

   (b) Given that $\sin\theta = \frac{5}{13}$ and θ is acute, use the identity to find the exact value of $\cos\theta$.

   (c) Hence, write down the exact value of $\tan\theta$.

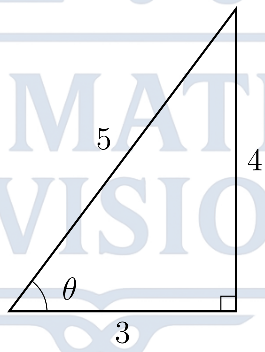
2. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
The graphs of $f(x) = 3\sin x$ and $g(x) = 2\cos x$ are shown below for $0 \leq x \leq 2\pi$.



(a) By equating $f(x)$ and $g(x)$, show algebraically that the points of intersection occur where $\tan x = \frac{2}{3}$.

(b) Use your graphic display calculator to find the exact x-coordinates of the two points of intersection shown in the graph.

3. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
A right-angled triangle has an angle θ such that $\cos\theta = \frac{3}{5}$.

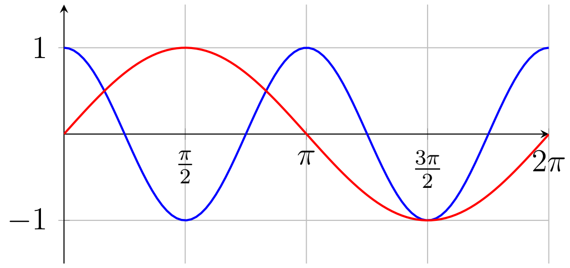


Using the double angle identities, find the exact values of:

(a) $\sin 2\theta$

(b) $\cos 2\theta$

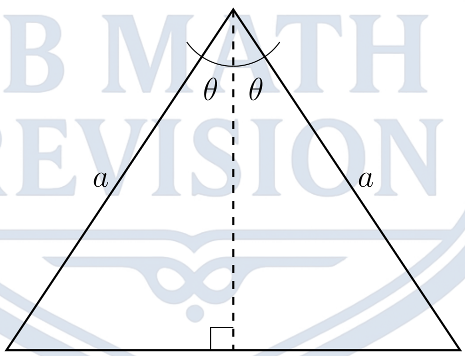
4. **[Paper 2 Style, Calculator Required, Easy, 4 marks]**
The diagram below shows the graphs of $y = \cos(2x)$ and $y = \sin(x)$ intersecting in the interval $0 \leq x \leq 2\pi$.



(a) Use the double angle identity for cosine to show that the intersections occur where $2\sin^2 x + \sin x - 1 = 0$.

(b) Use your GDC to find the values of x where the graphs intersect in this domain.

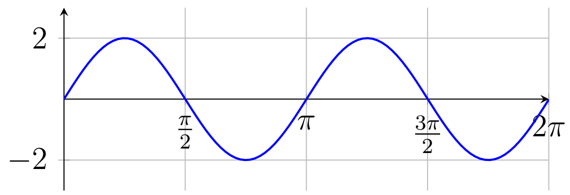
5. **[Paper 1 Style, Non-Calculator, Medium, 6 marks]**
Consider the isosceles triangle below with two sides of length a and an included angle of 2θ. The altitude (dashed line) divides the triangle into two right-angled triangles.



(a) Use right-angled trigonometry to find expressions for the base and the altitude of the entire triangle in terms of a and θ.

(b) By calculating the area of the triangle in two different ways, prove the double angle identity for sine: $\sin 2\theta = 2\sin\theta\cos\theta$.

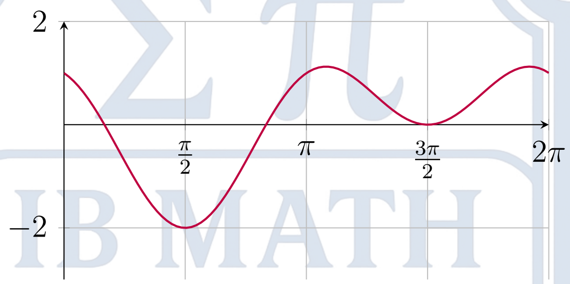
6. **[Paper 2 Style, Calculator Required, Medium, 4 marks]**
The function $f(x) = 4\sin x \cos x$ is plotted below.



(a) Use a double angle identity to write $f(x)$ in the form $A\sin(Bx)$.

(b) Use your GDC to find the exact coordinates of the first positive maximum point on the graph.

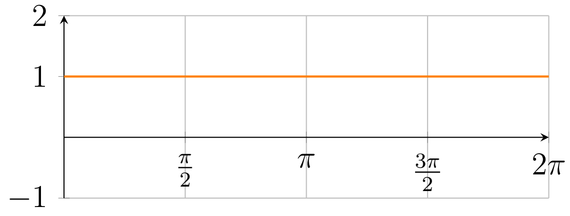
7. **[Paper 1 Style, Non-Calculator, Medium, 5 marks]**
The graph of $y = 2\cos^2 x - \sin x - 1$ is shown below. It intersects the x-axis at three points between 0 and 2π.



(a) Use the Pythagorean identity to express the equation $2\cos^2 x - \sin x - 1 = 0$ entirely in terms of $\sin x$.

(b) Hence, solve the equation analytically to find the exact x-coordinates of the three roots shown in the graph.

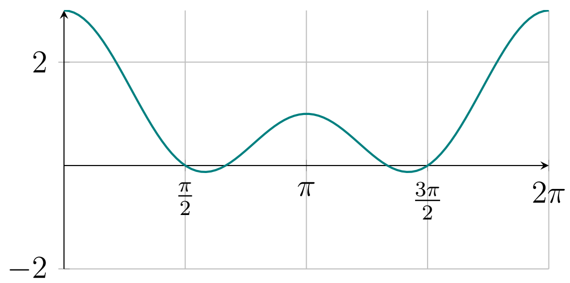
8. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
The height of a mechanical wave $h(t)$ at time t is modeled by $h(t) = \cos(2t) + 2\sin^2(t)$. The graph of this function is shown below.



(a) The graph appears to be a perfectly horizontal straight line. Write down the equation of this line.

(b) Using the double angle identity for cosine, prove algebraically that $h(t)$ is completely independent of t, confirming your answer to part (a).

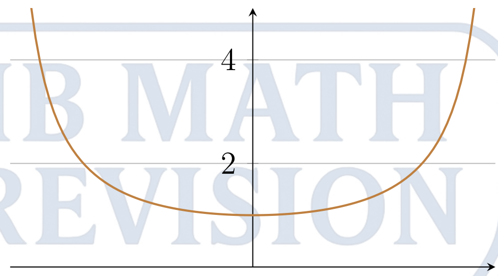
9. **[Paper 1 Style, Non-Calculator, Hard, 5 marks]**
Consider the trigonometric equation $\cos 2x + \cos x + 1 = 0$. The graph of the function $y = \cos 2x + \cos x + 1$ is shown below.



(a) Show that the equation can be rewritten as $\cos x(2\cos x + 1) = 0$.

(b) Find the exact values of the three x-intercepts shown on the graph for $0 \leq x \leq 2\pi$.

10. **[Paper 2 Style, Calculator Required, Hard, 5 marks]**
A function is defined as $f(x) = \tan x \sin x + \cos x$. The graph of $f(x)$ is plotted below alongside a vertical asymptote.



(a) Use the relationship $\tan x = \frac{\sin x}{\cos x}$ and the Pythagorean identity to prove algebraically that $f(x) = \frac{1}{\cos x}$.

(b) Use your GDC to find the exact coordinates of the minimum point of this curve in the domain $-\frac{\pi}{2} < x < \frac{\pi}{2}$.