IB MATHEMATICS AI HL
UNIT 2: FUNCTIONS

Advanced Modelling

Instructions to Candidates

- This question booklet contains **15 questions**.
- The paper targets **AHL** syllabus component 2.9.
- Answer all questions, showing all step-by-step working clearly.

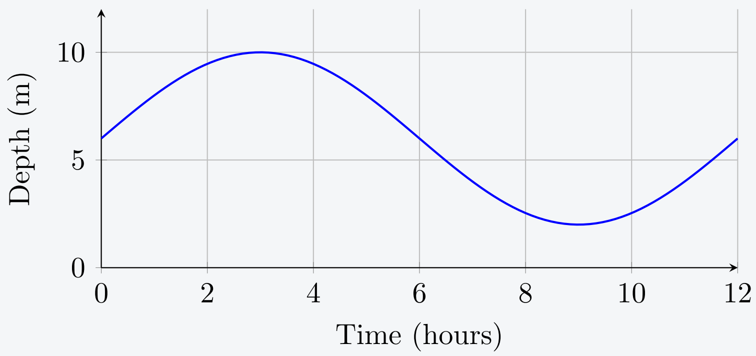
Difficulty Progression

- **Questions 1 - 5 (Easy):** Reading sinusoidal graphs, basic exponential decay, logistic carrying capacity, quadratic optimization.
- **Questions 6 - 10 (Medium):** Exponential half-life, continuous piecewise modelling, solving equations graphically.
- **Questions 11 - 15 (Hard):** Solving for multiple logistic parameters, calculating maximum rate of change (inflection points), and integrating between complex models.

SECTION A: EASY (Fundamentals)

Question 1 (4 Marks)

The depth of water, $D(t)$, in metres in a harbor is modelled by the following graph:



Based on the graph, state the equation of the principal axis and the amplitude. State the maximum and minimum depths of the water.

Question 2 (4 Marks)

A radioactive substance follows the decay model $M(t) = 200e^{-0.1t}$. Find the initial mass and the mass remaining after exactly 10 years.

Question 3 (4 Marks)

A population is modelled by the logistic curve $P(t) = \frac{5000}{1+24e^{-0.3t}}$. Write down the carrying capacity (maximum population limit) of this model.

Question 4 (4 Marks)

A quadratic cost model is given by $C(x) = 2x^2 - 16x + 50$. Find the number of units x that minimizes the cost.

Question 5 (5 Marks)

Find the period of the sinusoidal function $f(t) = 3\cos(\frac{\pi}{4}(t-2)) + 5$.

SECTION B: MEDIUM (Application & Modelling)

Question 6 (5 Marks)

For the logistic model $P(t) = \frac{5000}{1+24e^{-0.3t}}$, find the initial population at $t = 0$.

Question 7 (6 Marks)

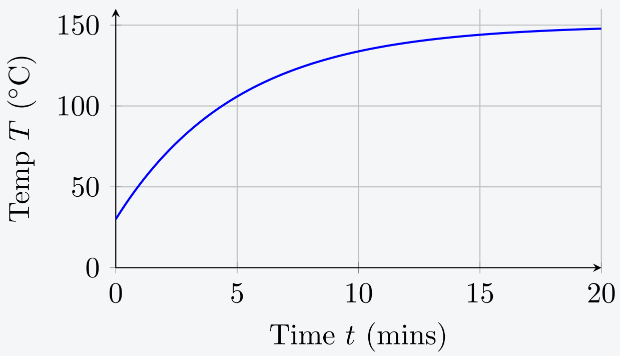
An isotope has a half-life of 15 hours. Assuming an exponential model $M(t) = M_0 e^{-kt}$, calculate the exact value of the decay constant k.

Question 8 (6 Marks)

A continuous piecewise function models a runner's speed: $S(t) = 5t$ for $0 \leq t \leq 4$, and $S(t) = at + b$ for $4 < t \leq 10$. If the runner's speed drops to 0 at $t = 10$, find the exact values of a and b.

Question 9 (6 Marks)

The temperature of an oven is modelled by the curve below:



The equation is $T(t) = 150 - 120e^{-0.2t}$. Find the exact time t when the temperature reaches 100°C.

Question 10 (6 Marks)

For the tide model $D(t) = 4\sin(\frac{\pi}{6}t) + 6$, find the first time $t > 0$ when the depth is exactly 8 metres.

SECTION C: HARD (Synthesis & Proof)

Question 11 (7 Marks)

The population of a species of insects in a closed environment is modelled by the logistic equation:

$$P(t) = \frac{L}{1 + Ce^{-kt}}$$

Initially, the population is 20. After 5 days, the population is 80. The environment can sustain a maximum of 500 insects.
Set up the necessary equations and algebraically solve for the exact values of L, C, and k.

Question 12 (8 Marks)

A Ferris wheel has a maximum height of 45m and a minimum height of 5m. It takes 30 minutes to complete one revolution. A rider boards at the bottom at $t = 0$.
Model their height with $h(t) = a\sin(b(t - c)) + d$. Find the exact parameters a, b, c, d, keeping in mind that the curve starts at a minimum, not on the principal axis.

Question 13 (8 Marks)

A disease spreads according to the logistic model $N(t) = \frac{2000}{1+19e^{-0.4t}}$.
The disease is spreading the fastest at the point of inflection of the curve.
(a) State the number of infected people when the disease is spreading the fastest. [**2 marks**]
(b) Use your GDC (or algebra) to find the exact time t when this maximum spread occurs. [**3 marks**]
(c) Calculate this maximum rate of spread in people per day. [**3 marks**]

Question 14 (7 Marks)

Two populations of bacteria are modelled by $P_A(t) = 100e^{0.2t}$ and $P_B(t) = 500e^{0.05t}$.
Find the exact time t when the populations are equal. Show your algebraic logarithmic steps.

Question 15 (9 Marks)

A city's power consumption in megawatts over 24 hours is modelled by $P(t) = 300 + 150\sin\left(\frac{\pi t}{12} - \frac{\pi}{2}\right)$.
Solar panels provide power modelled by $S(t) = 100 - 100\cos\left(\frac{\pi t}{12}\right)$.
Use the numerical integration feature on your GDC to calculate the total energy deficit (the area between the curves where $P(t) > S(t)$) over the full 24-hour period.