Topic 1: Laws of Logarithms & Exponential Models
IB Math AI SL

Answer all questions. Show all working where appropriate. Solutions found from a Graphic Display Calculator (GDC) should be supported by suitable working (e.g., sketching the graphs or stating the solver equations used). Total: 87 marks.

1. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
   Using the logarithm functions on your Graphic Display Calculator, evaluate the following expressions:

   (a) $\log_{10} 1000$

   (b) $\ln(e^4)$

   (c) $e^{\ln 7}$

2. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
   Use your GDC's equation solver or graphing tool to find the exact value of x for each of the following exponential equations. Give your answers correct to 3 significant figures.

   (a) $3^x = 20$

   (b) $e^{2x} = 18$

3. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
   The population of a rabbit colony, P, is modelled by the exponential function $P(t) = 1000 \times 2^{\frac{t}{3}}$, where t is the time in months since the colony was first observed.

   (a) Write down the initial population of the rabbit colony.

   (b) Calculate the population after exactly 9 months.

   (c) Using your GDC, find the exact time it takes for the population to reach 8000 rabbits.

4. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
A piece of industrial machinery depreciates in value according to the model $V(t) = 45\,000e^{-0.15t}$, where V is the value in dollars and t is the time in years since it was purchased.

(a) Calculate the value of the machinery exactly 4 years after purchase. Round your answer to the nearest dollar.

(b) Using your GDC, find the exact number of years it will take for the machinery's value to halve.

5. **[Paper 1 Style, Short Answer, Medium, 4 marks]**
Given that $p = \ln 2$ and $q = \ln 3$, use the laws of logarithms to express each of the following in terms of p and q:

(a) $\ln 6$

(b) $\ln 24$

6. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
Consider the two exponential functions $f(x) = e^x - 3$ and $g(x) = 1 - 3e^{-x}$.

(a) Sketch both graphs on your GDC.

(b) Using the intersection tool on your GDC, find the coordinates of the exact point(s) where the two graphs intersect.

7. **[Paper 1 Style, Short Answer, Hard, 5 marks]**
Natasha carries out an experiment on the growth of mould. She believes that the growth can be modelled by an exponential function $P(t) = Ae^{kt}$, where P is the area covered by mould in mm^2, t is the time in days since the start of the experiment, and A and k are constants. The area covered by mould is 112 mm^2 at the start of the experiment and 360 mm^2 after 5 days.

(a) Write down the value of A.

(b) Using your GDC's solver, find the value of k, giving your answer correct to 3 significant figures.

(c) Estimate the area covered by the mould after 8 days.

8. **[Paper 2 Style, Longer Question, Hard, 8 marks]**
The temperature of a cup of coffee, T in degrees Celsius, t minutes after being poured is given by the exponential model $T(t) = 20 + 75e^{-0.08t}$.

   (a) Find the initial temperature of the coffee.
   (b) Calculate the temperature of the coffee 10 minutes after it was poured.
   (c) Write down the equation of the horizontal asymptote of the graph of T, and state its meaning in the context of this problem.
   (d) Using your GDC, find the time it takes for the coffee to cool down to exactly 45°C.

9. **[Paper 1 Style, Short Answer, Hard, 6 marks]**
Consider the natural logarithmic function $g(x) = \ln(x - 2)$.

   (a) State the domain of $g(x)$.
   (b) State the equation of the vertical asymptote of $g(x)$.
   (c) Find the exact coordinates of the x-intercept.

10. **[Paper 1 Style, Short Answer, Hard, 5 marks]**
Consider the exponential equation $200e^{-0.05t} = 40$.

   (a) Use your GDC to solve for t, giving your answer correct to 3 decimal places.
   (b) Show algebraically how to rearrange this equation into the exact form $t = a \ln b$, where $a, b \in \mathbb{Z}^+$.

11. **[Paper 2 Style, Longer Question, Hard, 8 marks]**
A biologist is observing the growth of a new bacteria strain. She records the population, P, every hour, t.

Time, t (hours)	1	2	3	4	5
Population, P	12	35	100	285	800

The growth can be modelled by an exponential regression equation of the form $P = a \times b^t$.

   (a) Using the exponential regression feature on your GDC, find the values of a and b.
   (b) Write down the exponential regression equation.
   (c) Use your equation to estimate the population of the bacteria after 7 hours.

12. **[Paper 2 Style, Longer Question, Very Hard, 6 marks]**
The Richter scale measures the magnitude, M, of an earthquake based on the seismic wave intensity, I, using the base 10 logarithmic model:

$$M = \log_{10}\left(\frac{I}{S}\right)$$

where S is the intensity of a standard reference earthquake.

(a) If an earthquake has a magnitude of $M = 7.0$, find an expression for its intensity I in terms of S.

(b) Calculate how many times more intense an earthquake of magnitude 7.0 is compared to an earthquake of magnitude 4.0.

13. **[Paper 2 Style, Longer Question, Very Hard, 7 marks]**
The value of a vintage guitar, $V(t)$, in dollars, t years after the year 2010 is modelled by the function:

$$V(t) = 12\,870 - k(1.1)^t$$

where k is a constant. In the year 2010 ($t = 0$), the value of the guitar was \$9790.

(a) Find the value of k.

(b) Calculate the value of the guitar in the year 2012 ($t = 2$).

(c) The guitar's value continues to follow this model until it becomes completely worthless. Using your GDC, find the year in which the value of the guitar drops to zero.

14. **[Paper 1 Style, Short Answer, Very Hard, 5 marks]**
Consider the exponential equation $4^x - 3(2^x) - 10 = 0$.

(a) Enter the function $y = 4^x - 3(2^x) - 10$ into your GDC graphing tool.

(b) Using the root/zero finding feature of your GDC, find the exact solution for x.

(c) Explain briefly why there is only one real solution to this equation, despite it taking a quadratic structure.

15. **[Paper 2 Style, Longer Question, Very Hard, 10 marks]**
Two different vehicles are depreciating in value. Car A's value, V_A in dollars, t years after purchase is modelled by $V_A(t) = 25\,000(0.88)^t$. Car B's value, V_B in dollars, t years after purchase is modelled by $V_B(t) = 30\,000e^{-0.18t}$.

(a) Calculate the value of Car A and Car B after 2 years.

(b) Using your GDC, graph both models on the same set of axes. Find the exact time, t, when both cars have the exact same value.

(c) Calculate the financial drop in value (depreciation loss) for Car A specifically during its 3rd year (the difference in value between $t = 2$ and $t = 3$).

(d) Find the value of t when Car A is worth exactly double the value of Car B.