

IB MATHEMATICS AI HL

UNIT 4: STATISTICS

Combinations of Random Variables & Poisson

Instructions to Candidates

- This question booklet contains **15 questions**.
- The paper targets **AHL** syllabus components 4.14 and 4.15.
- Answer all questions, showing all step-by-step working clearly.

Difficulty Progression

- **Questions 1 - 5 (Easy):** Linear transformations $E(aX + b)$, basic Poisson probabilities, and Poisson variance properties.
- **Questions 6 - 10 (Medium):** Adding independent Poisson distributions, combinations of normal distributions, and finding expected profits.
- **Questions 11 - 15 (Hard):** The $Var(X_1 + X_2)$ vs $Var(2X)$ trap, Poisson conditional probability, and multi-component manufacturing (sum of normals).

SECTION A: EASY (Fundamentals)

Question 1 (4 Marks)

A random variable X has an expected value $E(X) = 8$ and a variance $Var(X) = 5$.
 A new variable is defined by the linear transformation $Y = 3X - 4$.
 Calculate $E(Y)$ and $Var(Y)$.

CG50 Tip: The Poisson Distribution

To find Poisson probabilities on your calculator, go to MENU 2 (Statistics) → DIST (F5) → POISSON. Use Ppd for exact values $P(X = k)$ and Pcd for cumulative values $P(X \leq k)$.

Question 2 (4 Marks)

The number of emails received by a small business in a one-hour period follows a Poisson distribution with a mean of 4.5.
 Find the probability that the business receives exactly 6 emails in a randomly chosen one-hour period.

Question 3 (4 Marks)

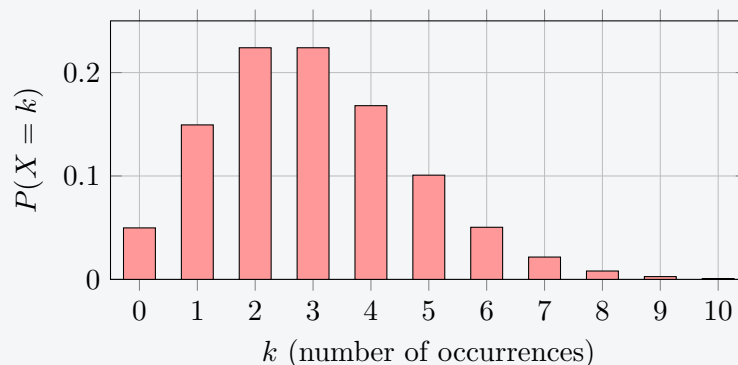
X and Y are independent random variables where: $E(X) = 12$, $Var(X) = 3$, $E(Y) = 7$, $Var(Y) = 2$
 Find the expected value and variance of the combination $X - Y$.

Question 4 (4 Marks)

A discrete random variable follows a Poisson distribution $X \sim Po(m)$.
 Given that $P(X = 1) = P(X = 2)$, algebraically find the exact value of the mean m .

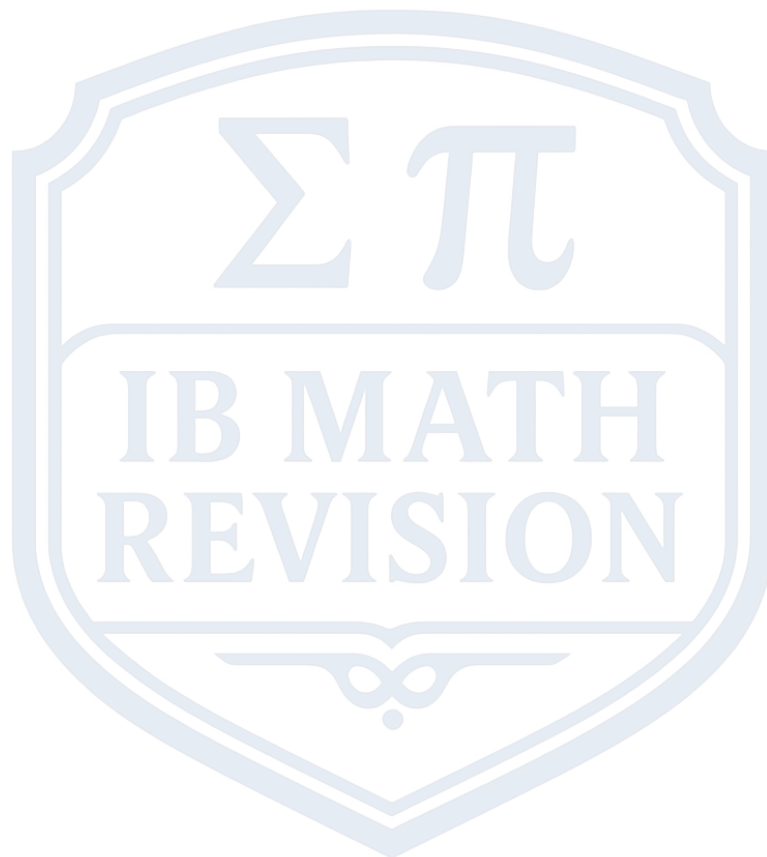
Question 5 (5 Marks)

The bar chart below represents a Poisson distribution $X \sim Po(3)$.



(a) Write down the exact variance of this distribution. [1 mark]

(b) Using the formula $P(X = k) = \frac{e^{-m} m^k}{k!}$, show algebraically why the bars for $k = 2$ and $k = 3$ are exactly the same height. [4 marks]



SECTION B: MEDIUM (Application & Modelling)

Question 6 (5 Marks)

The weights of apples are normally distributed with a mean of 150g and a standard deviation of 10g.

If two apples are chosen at random, find the probability that their combined weight is greater than 315g.

Question 7 (6 Marks)

On a quiet rural road, the number of cars passing per hour follows a Poisson distribution with mean 2.5. The number of trucks passing per hour follows an independent Poisson distribution with mean 1.2.

Find the probability that a total of exactly 5 vehicles (cars and trucks combined) pass by in a randomly chosen one-hour period.

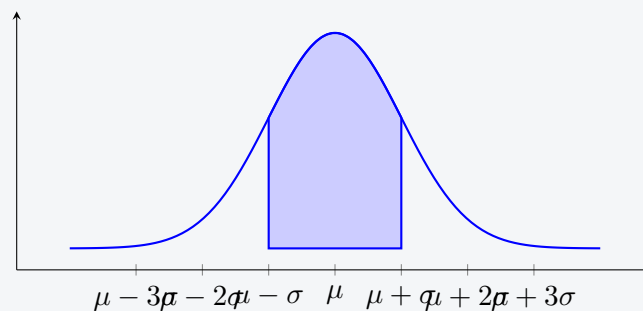
Question 8 (6 Marks)

A baker's daily profit on croissants, C , has a mean of \$45 and a variance of 16. The daily profit on muffins, M , has a mean of \$30 and a variance of 9.

Assume C and M are independent. The baker calculates their total daily profit as $T = 3C + 2M$. Find $E(T)$ and $Var(T)$.

Question 9 (6 Marks)

The diagram below shows a normal distribution curve for a random variable $X \sim N(\mu, \sigma^2)$. The shaded region represents $P(\mu - \sigma < X < \mu + \sigma)$.

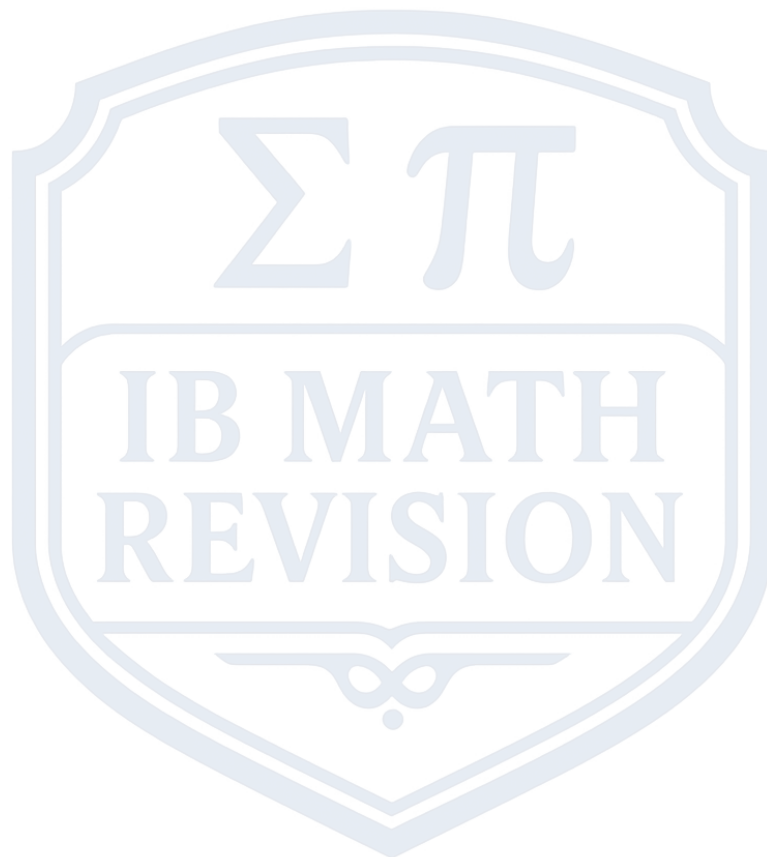


A manufacturer produces steel rods. The lengths of the rods, L_1 and L_2 , are independent and follow $N(50, 4)$.

Find the probability that the sum of the lengths of two randomly chosen rods, $L_1 + L_2$, falls strictly within one standard deviation of its combined mean.

Question 10 (6 Marks)

A factory machine breaks down at an average rate of 0.8 times per week. Assuming the breakdowns follow a Poisson distribution, calculate the probability that the machine breaks down exactly 4 times in a 4-week period.



SECTION C: HARD (Synthesis & Proof)**Question 11 (7 Marks)**

Let X be the volume of juice in a bottle, where $E(X) = 500\text{ml}$ and $Var(X) = 15$.

A student claims that taking one bottle and multiplying its volume by two, $Y = 2X$, has the exact same variance as taking two independent bottles and adding their volumes together, $W = X_1 + X_2$.

Determine algebraically whether the student is correct, and explain the mathematical reason for the difference in variance.

Question 12 (8 Marks)

The number of meteorites observed by an astronomer in an hour follows a Poisson distribution with $\lambda = 3.2$.

Given that the astronomer has already observed at least 2 meteorites in a specific hour, find the probability that they observed exactly 4 meteorites in that hour.

Question 13 (8 Marks)

A company ships items in boxes. The weight of an empty box is $B \sim N(200, 40^2)$ grams.

The company packs 12 identical, independent items into the box. The weight of each item is $I \sim N(250, 15^2)$ grams.

Let T be the total weight of the packed box. Calculate the exact standard deviation of T , and find $P(T < 3100)$.

Question 14 (7 Marks)

A factory manufactures wiring. The number of flaws in the wiring follows a Poisson distribution with an average of 2 flaws per 10 metres of wire.

If an electrician randomly selects a 35-metre roll of wire, find the probability that it contains fewer than 6 flaws.

Question 15 (9 Marks)

An elevator has a strict maximum weight limit of 450 kg.

5 people enter the elevator. The weight of a person in this building is normally distributed with a mean of 80 kg and a standard deviation of 10 kg.

(a) State the distribution of the total combined weight, W_{Total} , of the 5 independent people. [4 marks]

(b) Calculate the probability that the elevator's weight limit is exceeded. [5 marks]