Topic: Integration & Area Under Curves
IB Math AI SL

Answer all questions. Show all working where appropriate. Total: 85 marks.

1. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
Find the indefinite integral (anti-derivative) for the following polynomial functions:

   (a) $\int (6x^2 - 4x + 3)\, dx$

   (b) $\int \left(\frac{4}{x^3} + 2\right) dx$

2. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
Consider the function $f(x) = \frac{1}{2}x^2 + 2$. We wish to estimate the area under the curve between $x = 0$ and $x = 4$.

   (a) Copy and complete the following table of values.

x	0	1	2	3	4
$f(x)$	2		4		10

   (b) Use the trapezoidal rule with $h = 1$ to find an approximation for $\int_0^4 \left(\frac{1}{2}x^2 + 2\right) dx$.

3. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
Refer to the function $f(x) = \frac{1}{2}x^2 + 2$ from Question 2.

   (a) Using your graphic display calculator, calculate the exact value of the definite integral $\int_0^4 \left(\frac{1}{2}x^2 + 2\right) dx$.

   (b) Calculate the percentage error between your trapezoidal approximation in Question 2 and the exact value.

4. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
A curve has a gradient function given by $\frac{dy}{dx} = 3x^2 - 8x$. The curve passes through the point $(2, 5)$.

   (a) Find the general expression for y by integrating the gradient function.

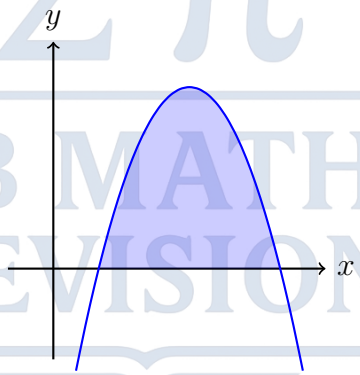
   (b) Use the given point to find the constant of integration, C, and write down the exact equation of the curve.

5. **[Paper 2 Style, Longer Question, Medium, 6 marks]**
The curve $y = -x^2 + 6x - 5$ forms an enclosed region with the x-axis.

   (a) Use your graphic display calculator to find the x-intercepts (roots) of the curve.

   (b) Write down the definite integral that represents the area of the enclosed region.

   (c) Evaluate the area of this region.



6. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
A car's velocity, v in m s^{-1}, is measured at 5-second intervals. The data is recorded in the table below.

Time (t seconds)	0	5	10	15	20
Velocity (v)	12	18	22	24	20

   (a) Write down the value of the interval width, h, for this data.

   (b) Use the trapezoidal rule to estimate the total distance travelled by the car during the 20 seconds.

7. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
The velocity of a particle, v in m s^{-1}, is given by $v(t) = t^2 - 7t + 10$ for $t \geq 0$.

   (a) Determine the times at which the particle is momentarily at rest.

   (b) Use your graphic display calculator to find the total distance travelled by the particle in the first 6 seconds.

8. **[Paper 2 Style, Longer Question, Medium, 6 marks]**
A company's marginal cost function, which represents the rate of change of the total cost $C(x)$ to produce x units, is $C'(x) = 0.6x^2 - 4x + 20$. The fixed costs (the cost when $x = 0$) are \$150.

   (a) Find the expression for the total cost function, $C(x)$.

   (b) Calculate the total cost of producing exactly 10 units.

   (c) Find the exact cost of producing the 10th unit (the cost increase from 9 units to 10 units).

9. **[Paper 1 Style, Short Answer, Hard, 5 marks]**
Find the exact value of the constant k ($k > 0$) such that:

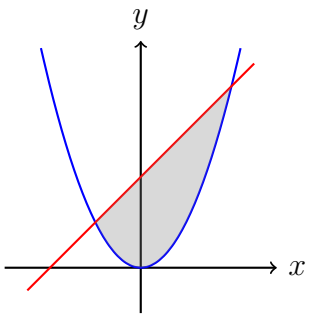
$$\int_0^k 3x^2 \, dx = 64$$

10. **[Paper 2 Style, Longer Question, Hard, 7 marks]**
The graphs of $y = x^2$ and $y = x + 2$ enclose a region.

   (a) Use your graphic display calculator to find the x-coordinates of the points of intersection of the two graphs.

   (b) Write down the definite integral that represents the area of the enclosed region.

   (c) Calculate the exact area of this region.



11. **[Paper 1 Style, Short Answer, Hard, 5 marks]**
A solar panel generates electricity at a rate given by $P(t) = 150\sin(0.3t)$, where P is the power in Watts, and t is the time in hours after sunrise. The total energy generated is the area under the power-time graph.

    (a) Write down the definite integral to find the total energy generated in the first 8 hours.

    (b) Using your graphic display calculator, evaluate this integral.

12. **[Paper 2 Style, Longer Question, Hard, 6 marks]**
The curve $y = x^3 - 4x$ crosses the x-axis at three points, creating two bounded regions between the curve and the x-axis.

    (a) Find the x-intercepts of the curve.

    (b) Explain why calculating $\int_{-2}^{2}(x^3 - 4x)\,dx$ will not give the total area enclosed by the curve and the x-axis.

    (c) Use your graphic display calculator to find the total area of the two enclosed regions.

13. **[Paper 2 Style, Longer Question, Very Hard, 7 marks]**
The area enclosed by the curve $y = \frac{1}{x^2}$ $(x > 0)$, the x-axis, the line $x = 1$, and the line $x = a$ (where $a > 1$) is exactly 0.8 square units.

    (a) Find an expression for $\int_1^a \frac{1}{x^2}\,dx$ in terms of a.

    (b) Set up an equation and solve it to find the exact value of a.

    (c) Verify your answer using the integral solver on your graphic display calculator.

14. **[Paper 1 Style, Short Answer, Very Hard, 5 marks]**
Water is leaking from a tank at a rate of $R(t) = 50e^{-0.2t}$ litres per minute.

    (a) Calculate the total volume of water that leaks out of the tank in the first 10 minutes.

    (b) Determine the limit of the total volume leaked as $t \to \infty$.

15. **[Paper 2 Style, Longer Question, Very Hard, 8 marks]**
Consider the curves $f(x) = 5 - x^2$ and $g(x) = e^{0.5x}$.

(a) Graph both functions on your graphic display calculator and find the x-coordinates of their points of intersection.

(b) Write down the integral required to find the area of the region completely enclosed by the two curves.

(c) Use the `G-Solv` integration feature to find the total area of this enclosed region.