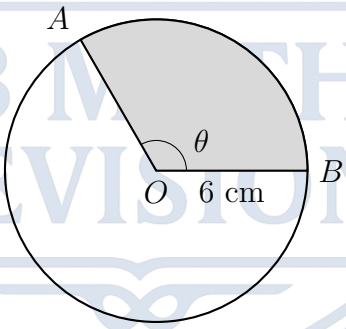
Unit 3: Radians, Arc Length & Sectors
IB Math AA SL

Answer all questions. Show all working where appropriate. For Paper 1 questions, you must use analytical algebraic methods and give exact answers. For Paper 2 questions, use your graphic display calculator (GDC) efficiently and round to 3 significant figures.

1. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
The following diagram shows a circle with centre O and radius 6 cm. Points A and B lie on the circumference and angle $AOB = \theta$ radians.



The perimeter of the shaded sector OAB is exactly 30 cm.

(a) Find the exact value of θ.

(b) Hence, find the exact area of the shaded sector OAB.

2. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
A lawn sprinkler sprays water over a lawn covering an arc of 1.8 radians with a maximum spray distance of r metres. The lawn sprinkler perfectly waters a sector of area 20 m^2 of the lawn.

(a) Calculate the value of the maximum spray distance, r.

(b) Calculate the length of the outer arc watered by the sprinkler.

3. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
A sector of a circle OPQ has a radius of 4 cm and an internal angle of $\frac{\pi}{3}$ radians.

   (a) Find the exact area of the triangle OPQ.

   (b) Hence, find the exact area of the minor segment enclosed by the chord PQ and the arc PQ.

4. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
A windscreen wiper blade is 0.8 m long. When in motion, the blade moves through an arc of θ radians and wipes an area of 0.6 m^2 on the glass.

   (a) Calculate the value of θ.

   (b) Calculate the total distance travelled by the outer edge of the wiper blade during one single sweep.

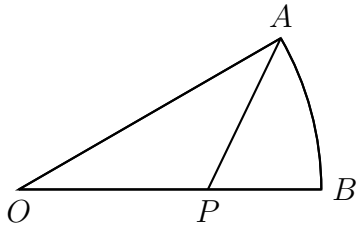
5. **[Paper 1 Style, Non-Calculator, Medium, 5 marks]**
Consider a sector of a circle with radius R cm and an angle of θ radians at the centre. The area of the sector (in cm^2) is numerically equal to exactly three times the length of its arc (in cm). Find the exact value of R.

6. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
A circle has a radius of 12 cm. A straight chord AB is drawn across the circle such that the length of the chord is 18 cm.

   (a) Use the cosine rule (or right-angled trigonometry) to find the angle θ subtended by the chord at the centre of the circle, giving your answer in radians.

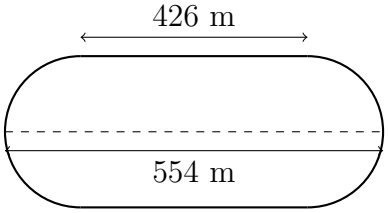
   (b) Hence, calculate the area of the minor segment bounded by the chord AB and the arc AB.

7. **[Paper 1 Style, Non-Calculator, Medium, 6 marks]**
The diagram below shows the sector of a circle OAB with centre O and radius r. The angle at the centre of the sector, AOB, is $\frac{\pi}{6}$ radians. Point P is located on the line segment $[OB]$ such that $OP = k \times OB$, where k is a constant with $0 < k < 1$.



Given that the exact area of the triangle OAP is exactly half the area of the sector OAB, find the exact value of k.

8. **[Paper 2 Style, Calculator Required, Medium, 6 marks]**
The diagram below shows a dirt racetrack where the two straights are exactly 426 m long and are parallel to each other. The ends of the track are perfect semi-circles. The longest distance from one end of the track to the other end (measured across the semi-circles) is 554 m.



   (a) Find the total distance around the entire perimeter of the racetrack.

   (b) Find the total area enclosed by the racetrack.

9. **[Paper 1 Style, Non-Calculator, Hard, 6 marks]**
The diagram below shows the sector of a circle OAB with centre O. The angle at the centre of the sector, AOB, is $\frac{5\pi}{6}$ radians. Point M is the exact midpoint of line segment $[OA]$. Show that the ratio of the area of triangle OMB to the area of the entire sector OAB can be expressed in the form $3 : p\pi$, where $p \in \mathbb{Z}^+$.

10. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
A games design company produces solid plastic game pieces in the form of a right prism. The uniform cross-section of the prism is the sector of a circle. The angle at the centre of the sector is 1.05 radians, and the uniform height of the game piece is 4 mm.

    (a) Given that the total volume of the game piece is 412 mm^3, calculate the radius of the sector cross-section.

    (b) Hence, calculate the total surface area of the plastic game piece in mm^2.

11. **[Paper 1 Style, Non-Calculator, Hard, 7 marks]**
OAB is a sector of a circle with centre O and radius r. The angle AOB is θ radians, where $0 < \theta < \frac{\pi}{2}$. A point C lies on the line segment $[OA]$ such that $[OA]$ is perpendicular to $[BC]$.

    (a) Show that $OC = r\cos\theta$ and $BC = r\sin\theta$.

    (b) Show that the exact area of the triangle OBC can be written as $\frac{1}{4}r^2\sin(2\theta)$.

    (c) Given that the area of the triangle OBC is equal to exactly $\frac{1}{\pi}$ of the area of the sector OAB, show that $\pi\sin(2\theta) = 2\theta$.

12. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
A straight chord AB divides a circle of radius 8 cm into two distinct segments. The area of the minor segment is exactly 25 cm^2.

(a) Let θ be the angle subtended by the chord AB at the centre of the circle, measured in radians. Write down an equation for the area of the minor segment in terms of θ.

(b) Using the equation solver on your graphic display calculator, find the value of θ.

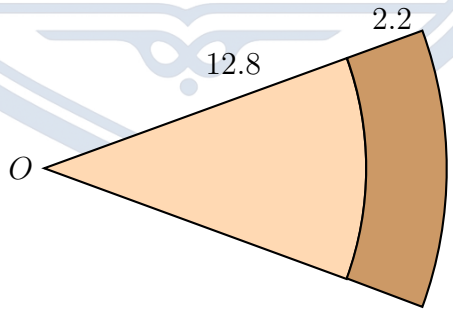
(c) Hence, find the length of the chord AB.

13. **[Paper 1 Style, Non-Calculator, Very Hard, 7 marks]**
A three-dimensional right circular cone is formed by taking a 2D circle of radius R, cutting out a sector with an internal angle of $\frac{4\pi}{3}$ radians, and seamlessly joining the two straight straight radii edges of that sector together.

(a) Show that the exact base radius of the resulting cone, r, is equal to $\frac{2}{3}R$.

(b) Hence, find the exact ratio of the cone's vertical height, h, to its slant height, R. Give your answer in the form $\frac{\sqrt{a}}{b}$ where $a, b \in \mathbb{Z}^+$.

14. **[Paper 2 Style, Calculator Required, Very Hard, 7 marks]**
The diagram below shows a slice of pizza that forms a sector of a circle with an angle of 40° and a total radius of 15 cm. The crust forms a uniform band along the outer curved edge with a width of 2.2 cm.



(a) Convert the angle of the pizza slice into radians.

(b) Calculate the exact area of the crust of the pizza slice in cm^2.

(c) Calculate the total perimeter of the crust itself (the shaded brown region).

15. **[Paper 1 Style, Non-Calculator, Very Hard, 8 marks]**
A sector of a circle OAB has a centre O, radius r, and an internal angle θ radians. A smaller circle of radius R is perfectly inscribed inside the sector such that it is tangent to the line segment $[OA]$, the line segment $[OB]$, and the arc AB. By drawing a line from O through the centre of the inscribed circle, C, show that the exact radius of the inscribed circle is given by:

$$R = \frac{r \sin\left(\frac{\theta}{2}\right)}{1 + \sin\left(\frac{\theta}{2}\right)}$$