Unit 1: Financial Applications (Compound Interest & Depreciation)
IB Math AA SL

Answer all questions. Show all working where appropriate. Use your graphic display calculator (GDC) to efficiently solve or verify equations where possible.

1. **[Paper 2 Style, Short Answer, Easy, 6 marks]**
   On 1st January 2021, Nerys invests $\$P$ in an account that pays a nominal annual interest rate of 4.2%, compounded monthly. The amount of money in Nerys' account at the end of each year follows a geometric sequence with common ratio, r.

   (a) Find the exact value of r, giving your answer to four decimal places.

   (b) Nerys makes no further deposits to or withdrawals from the account. Find the year in which the amount of money in Nerys' account will become double the amount she invested.

2. **[Paper 2 Style, Short Answer, Easy, 8 marks]**
   On his 40th birthday, Robert invests \$15 000 into a savings account that pays a nominal annual interest rate of 4.78%, compounded monthly.

   (a) Write an expression for the total value of the investment after n years. Give your numerical values to 5 decimal places.

   (b) Find the total amount in the savings account after 3 years and 5 years.

   (c) Find the exact age Robert will be when the amount of money in his account is 1.5 times the initial amount.

3. **[Paper 2 Style, Short Answer, Easy, 3 marks]**
   Daniel invests \$5000 in an account that pays a nominal annual interest rate of 2.24%, compounded quarterly. Calculate the amount Daniel will have in his account after 8 years. Give your answer to 2 decimal places.

4. **[Paper 2 Style, Short Answer, Easy, 3 marks]**
Jonah wants to invest his money in an account such that his \$5000 investment will double in exactly 10 years. Assume the account pays a nominal annual interest rate of $r\%$, compounded half-yearly. Determine the value of r.

5. **[Paper 2 Style, Short Answer, Medium, 5 marks]**
Amelia sets a target of saving \$20 000. She invests her initial \$9000 in an account that offers an interest rate of 7% per annum compounded annually.

   (a) Find the value of Amelia's investment after 5 years to the nearest hundred dollars.

   (b) Determine the number of years required for Amelia's investment to reach the target.

6. **[Paper 2 Style, Short Answer, Medium, 3 marks]**
Bill also sets a target of saving \$20 000. He invests his initial \$9000 in an account that offers an interest rate of $r\%$ per annum compounded monthly, where r is set to two decimal places. Find the minimum value of r needed for Bill to reach the target after exactly 10 years.

7. **[Paper 2 Style, Short Answer, Medium, 5 marks]**
Aneirin is given a gift of 12 000 by his grandmother on his twenty-first birthday. He invests his 12 000 in a bank account that offers an interest rate of 5.84% per annum compounded annually.

   (a) Find the value of Aneirin's investment after 9 years, to the nearest pound.

   (b) Determine the number of years it would take for the value of Aneirin's investment to double.

8. **[Paper 2 Style, Short Answer, Medium, 3 marks]**
Bran also receives 12 000 and invests it in a bank account that offers an interest rate of $r\%$ per annum, compounded monthly, where r is set to two decimal places. Find the minimum value of r needed if Bran is to have at least as much money in his account after 9 years as Aneirin has in his (from Question 7).

9. **[Paper 2 Style, Longer Question, Medium, 6 marks]**
Finn borrows \$3200 from his parents to buy a car. He agrees a repayment plan where he pays them back \$$c$ in the first month and then \$$m$ each subsequent month. After two months Finn has paid back his parents a total of \$1000, which can be expressed as $m + c = 1000$.

   (a) After half a year (6 months) he still owes his parents \$1000. Write down a second equation connecting m and c.

   (b) Solve the system of equations to find the value of m and c.

10. **[Paper 2 Style, Short Answer, Hard, 4 marks]**
Finn's parents decide to apply a 6.25% net interest charge to the initial \$3200 total before any repayments begin. Using your values of m and c from Question 9, calculate the total number of months it will take Finn to fully clear his debt.

11. **[Paper 2 Style, Longer Question, Hard, 6 marks]**
On 1st January 2020, Laurie invests \$$P$ in an account that pays a nominal annual interest rate of 5.5%, compounded quarterly.

   (a) Find the effective annual multiplier (the common ratio r for the year).

   (b) Laurie makes no further deposits to or withdrawals from the account. Using your GDC's equation solver or financial app, find the exact year and month in which the amount of money in Laurie's account will become double the amount she invested.

12. **[Paper 2 Style, Longer Question, Hard, 7 marks]**
Georgia buys a new computer for herself that costs \$1099. At the same time, she buys her son, Duncan, a new gaming computer that costs \$2749. It is anticipated that Georgia's computer will depreciate at a rate of 11% per year, whereas Duncan's gaming computer will depreciate at 18% per year.

   (a) Estimate the value of Georgia's computer after exactly 6 years.

   (b) On the same set of axes, sketch the depreciation curves for both computers. Clearly label the y-intercepts.

   (c) Find the exact time, in years, when Duncan's computer becomes worth less than Georgia's computer.

13. [**Paper 2 Style, Extended Question, Hard, 8 marks**]
Chris wants to reach a \$20 000 savings target. He does not trust banks, so he puts his money in a safe where it does not earn any interest. His system is to add money to this safe each year. Each year he will add exactly half the amount he added in the previous year.

    (a) Show that Chris will never reach the \$20 000 target if his initial deposit into the safe is \$9000.

    (b) Find the amount Chris needs to deposit initially in order to reach the \$20 000 target after exactly 5 years. Give your answer to the nearest dollar.

14. [**Paper 2 Style, Extended Question, Very Hard, 8 marks**]
Culhwch also puts his initial 12 000 in a metal box buried in his garden where he does not earn any interest. His savings plan is to add more money to the box each year, such that the money added each year will be a fixed multiple of the money added the previous year.

    (a) Given that each year Culhwch adds one third the amount he added in the previous year, show that his total savings will never reach 20 000.

    (b) Determine the minimum fixed multiplier he would need to use in order for his total savings to reach exactly 20 000 by the end of 9 years. Give your multiplier correct to 3 significant figures.

15. [**Paper 2 Style, Extended Question, Very Hard, 8 marks**]
Lucy is considering two investment strategies.
Strategy A requires an initial deposit of \$100. At the start of the second month a deposit of \$115 is made, with monthly deposits at the start of each month thereafter being \$15 more than the deposit in the previous month.
Strategy B requires an initial deposit of \$90. At the start of the second month a deposit of \$93.60 is made, with monthly deposits at the start of each month thereafter being 4% more than the deposit in the previous month.

    (a) Write an expression, using sigma notation, to represent the total amount invested after n months in Strategy A and Strategy B.

    (b) Use the graphing features of your GDC to find which monthly deposit from Strategy A would be the last one that is greater than the corresponding monthly deposit from Strategy B.

    (c) Find after which monthly deposit the total cumulative amount invested in Strategy B would finally exceed the total cumulative amount invested in Strategy A.