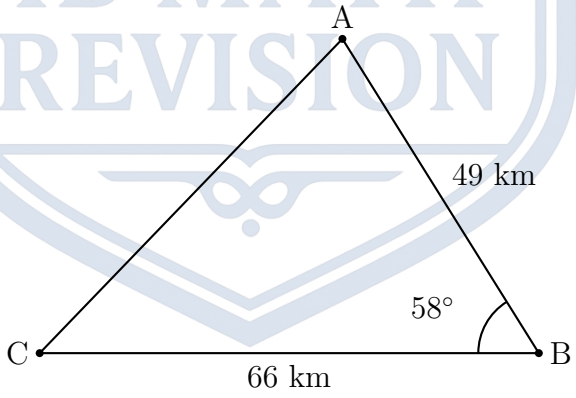
Topic: Non-Right-Angled Trigonometry
IB Math AI SL

Answer all questions. Show all working where appropriate. Total: 93 marks.

1. **[Short Answer, Easy, 5 marks]**
   A national park is in the shape of a triangle ABC. The length of side AB is 49 km and the length of side BC is 66 km. The angle $A\hat{B}C$ is 58°.

   (a) Calculate the length of the side AC.

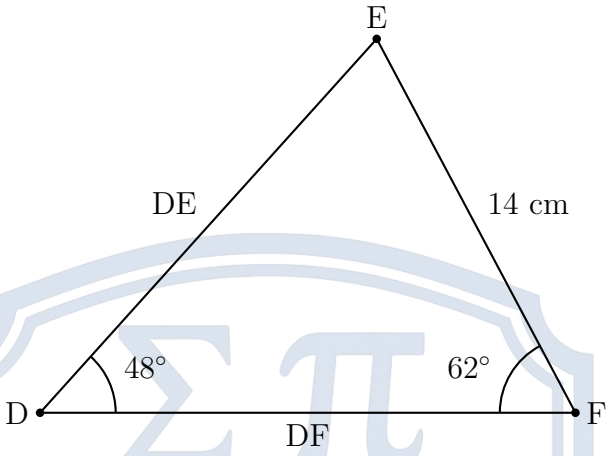
   (b) Calculate the total area of the national park.



2. **[Short Answer, Easy, 5 marks]**
In triangle DEF, the angle $E\hat{D}F = 48°$ and the angle $E\hat{F}D = 62°$. The length of the side $EF = 14$ cm.

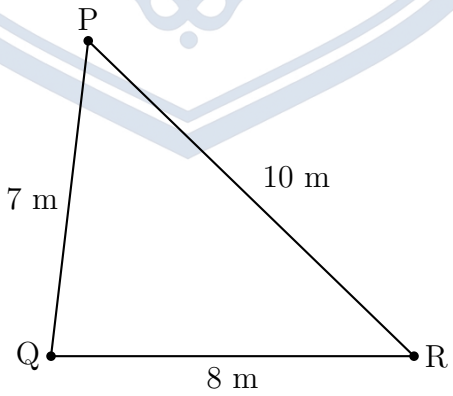
   (a) Find the size of the angle $D\hat{E}F$.

   (b) Calculate the length of side DF.



3. **[Short Answer, Easy, 4 marks]**
A triangular garden bed PQR has side lengths $PQ = 7$ m, $QR = 8$ m, and $PR = 10$ m.

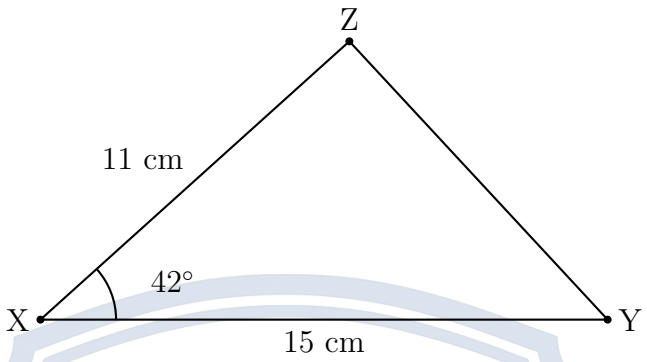
   (a) Find the size of the angle $P\hat{Q}R$.



4. **[Short Answer, Medium, 5 marks]**
In triangle XYZ, $XY = 15$ cm, $XZ = 11$ cm, and the included angle $Y\hat{X}Z = 42°$.

   (a) Calculate the length of YZ.

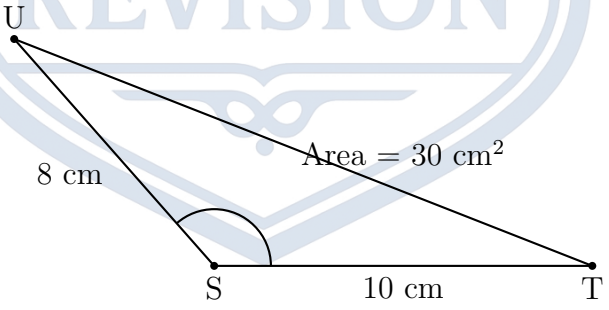
   (b) Find the size of the acute angle $X\hat{Z}Y$.



5. **[Short Answer, Medium, 6 marks]**
The area of triangle STU is 30 cm^2. The length $ST = 10$ cm and $SU = 8$ cm.

   (a) Find the two possible values for the angle $T\hat{S}U$.

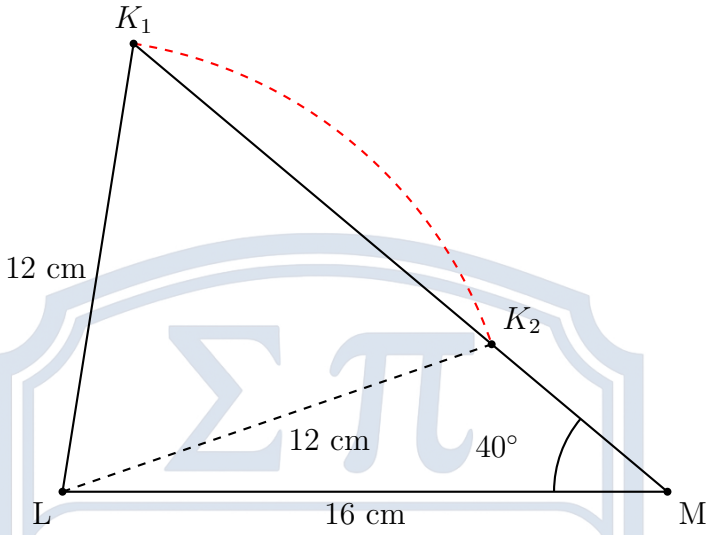
   (b) Given that $T\hat{S}U$ is an obtuse angle, calculate the length of TU.



6. **[Short Answer, Medium, 5 marks]**
Consider triangle KLM where $KL = 12$ cm, $LM = 16$ cm, and the angle $K\hat{M}L = 40°$.

   (a) Use the sine rule to find the two possible values for the angle $L\hat{K}M$.

   (b) Explain geometrically why there are two possible values (the ambiguous case).

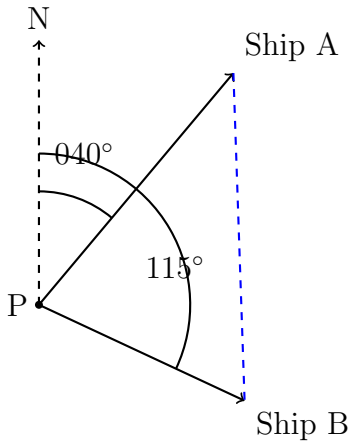


7. **[Longer Question, Medium, 7 marks]**
Two ships leave a port P at exactly the same time. Ship A travels at a constant speed of 20 km/h on a bearing of 040°. Ship B travels at a constant speed of 15 km/h on a bearing of 115°.

   (a) Calculate the distance each ship has travelled after 2 hours.

   (b) Find the angle between the two ships' paths ($A\hat{P}B$).

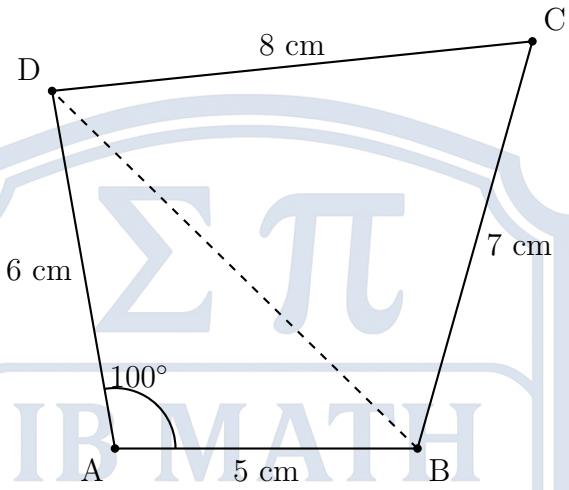
   (c) Calculate the direct straight-line distance between Ship A and Ship B after 2 hours.



8. **[Longer Question, Hard, 7 marks]**
A quadrilateral ABCD is formed by two adjacent triangles, ABD and BCD. The side lengths are $AB = 5$ cm, $BC = 7$ cm, $CD = 8$ cm, and $AD = 6$ cm. The angle $D\hat{A}B = 100°$.

(a) Calculate the length of the diagonal BD.

(b) Calculate the size of the angle $B\hat{C}D$.

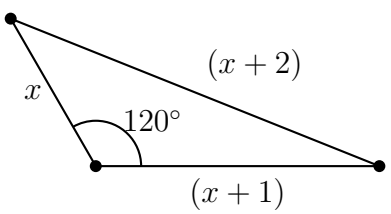


9. **[Longer Question, Hard, 8 marks]**
Consider a triangle with side lengths measuring x cm, $(x + 1)$ cm, and $(x + 2)$ cm. The largest angle in the triangle measures exactly 120°.

(a) Write down an expression for the side opposite the 120° angle.

(b) By using the cosine rule, form an equation in terms of x and show that it simplifies algebraically to $2x^2 - x - 3 = 0$.

(c) Hence, find the exact length of the shortest side of the triangle.

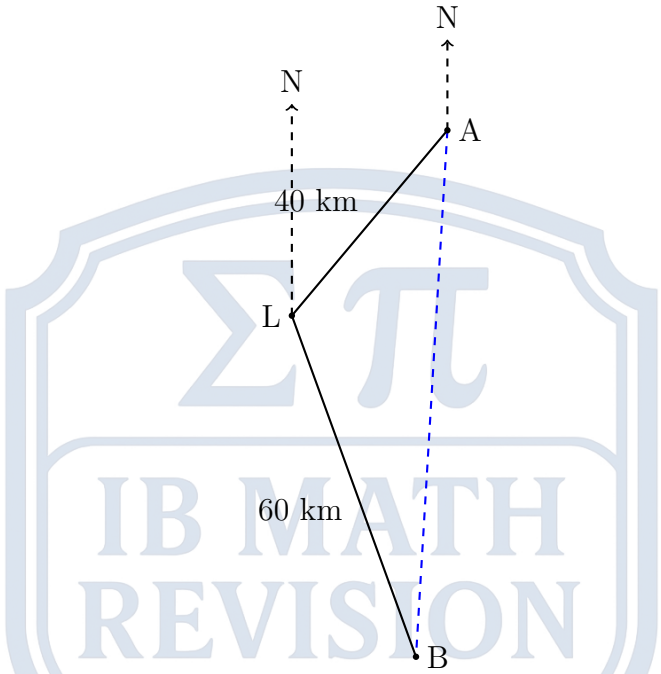


10. **[Longer Question, Hard, 8 marks]**
Ship A is 40 km from a lighthouse L on a bearing of 040°. Ship B is 60 km from the same lighthouse L on a bearing of 160°.

(a) Calculate the straight-line distance between Ship A and Ship B.

(b) Find the internal angle $L\hat{A}B$ within the triangle LAB.

(c) Hence, calculate the true three-figure bearing of Ship B from Ship A.

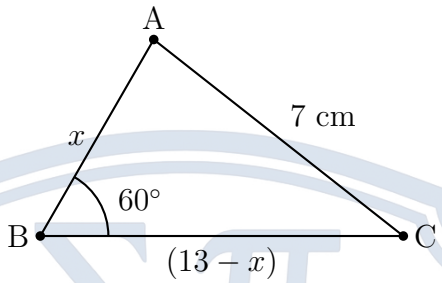


11. **[Longer Question, Hard, 7 marks]**
In triangle ABC, the perimeter is 20 cm. The side $AC = 7$ cm and the angle $A\hat{B}C = 60°$. Let the length of side AB be x cm.

(a) Write down an expression for the length of side BC in terms of x.

(b) Use the cosine rule to set up an equation and show that it simplifies to $x^2 - 13x + 40 = 0$.

(c) Given that AB is strictly shorter than BC, find the exact length of AB.

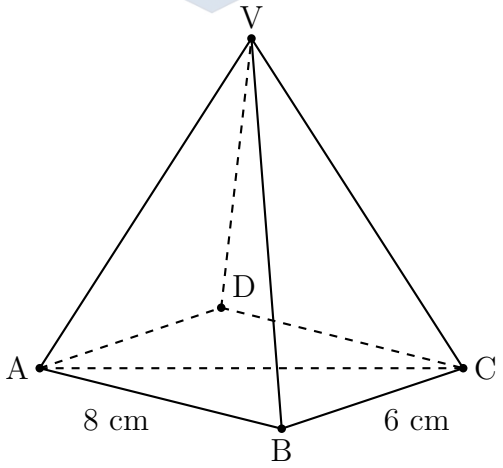


12. **[Longer Question, Hard, 7 marks]**
A right rectangular-based pyramid VABCD has a base where $AB = 8$ cm and $BC = 6$ cm. The four slant edges VA, VB, VC, and VD are each 12 cm long. The apex of the pyramid is V, and M is the midpoint of the side BC.

(a) Calculate the length of the base diagonal AC.

(b) Calculate the angle $V\hat{C}A$ between the slant edge VC and the base diagonal AC.

(c) Calculate the length of the line segment VM on the triangular face VBC.

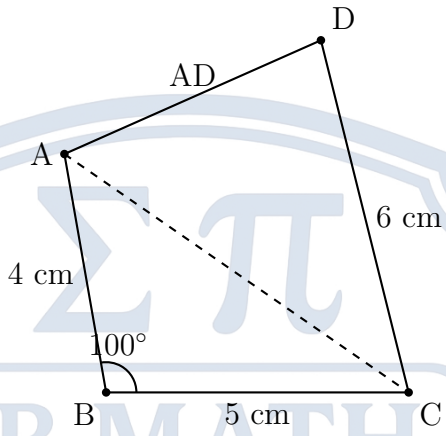


13. [**Longer Question, Very Hard, 8 marks**]
The points A, B, C, and D all lie on the circumference of a circle, forming a cyclic quadrilateral ABCD. The side lengths are $AB = 4$ cm, $BC = 5$ cm, and $CD = 6$ cm. The internal angle $A\hat{B}C = 100°$.

(a) State the size of the angle $A\hat{D}C$.

(b) Calculate the length of the diagonal AC.

(c) By considering triangle ADC, use the cosine rule to set up a quadratic equation and find the length of the side AD.



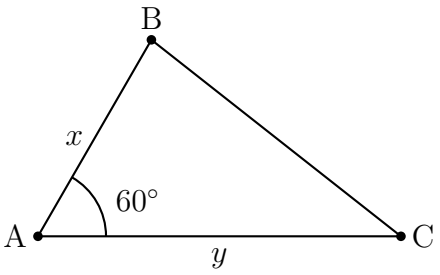
14. [**Longer Question, Very Hard, 9 marks**]
A triangle ABC has an area of $10\sqrt{3}$ cm^2 and a total perimeter of 20 cm. The angle $B\hat{A}C = 60°$. Let $AB = x$ cm and $AC = y$ cm.

(a) Use the area formula to show that $x \times y = 40$.

(b) Use the cosine rule to show that $BC^2 = (x + y)^2 - 120$.

(c) By writing an expression for BC in terms of the perimeter, find the exact value of $(x + y)$.

(d) Hence, determine the lengths of all three sides of the triangle ABC.



15. [**Longer Question, Very Hard, 5 marks**]
The Bermuda Triangle is a region of the Atlantic Ocean with Miami (M), Bermuda (B), and San Juan (S) as vertices. The distance between Miami and Bermuda is 1670 km, Bermuda and San Juan is 1550 km, and San Juan and Miami is 1660 km.

(a) Calculate the value of θ, the measure of angle $M\hat{S}B$.

(b) Calculate the total area of the Bermuda Triangle in km^2.