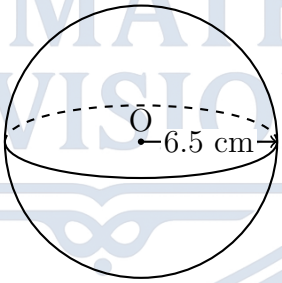
Topic: Volume and Surface Area of 3D Solids
IB Math AI SL

Answer all questions. Show all working where appropriate. Total: 93 marks.

1. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
A solid sphere made of steel has a radius of 6.5 cm.

   (a) Calculate the exact surface area of the sphere, in terms of π.

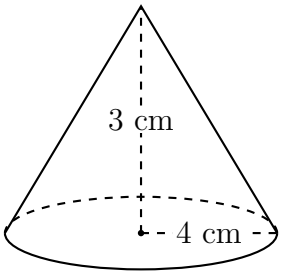
   (b) Calculate the volume of the sphere. Give your answer correct to 3 significant figures.



2. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
A solid right circular cone has a base radius of 4 cm and a vertical height of 3 cm.

   (a) Calculate the slant height of the cone.

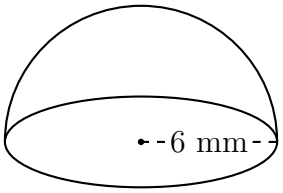
   (b) Calculate the curved surface area of the cone.



3. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
A piece of candy is made in the shape of a solid hemisphere. The radius of the hemisphere is 6 mm.

   (a) Calculate the total surface area of one piece of candy.

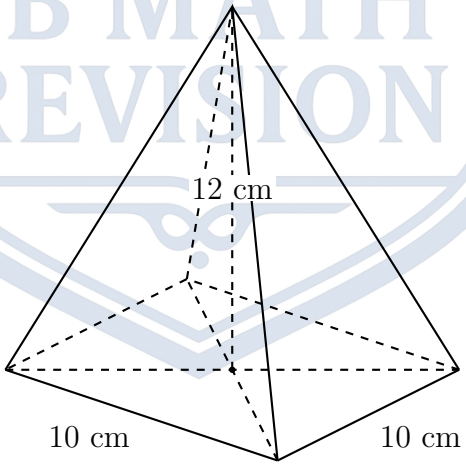
   (b) Calculate the volume of one piece of candy.



4. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
A right square-based pyramid has a base side length of 10 cm and a vertical height of 12 cm.

   (a) Calculate the slant height of one of the triangular faces.

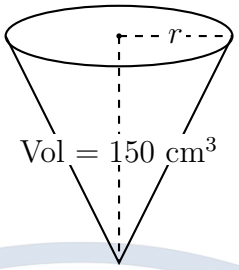
   (b) Hence, calculate the total surface area of the pyramid.



5. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
An ice cream cup is designed in the shape of a right circular cone. It has a volume of 150 cm^3 and a vertical height of 14 cm.

   (a) Find the radius of the circular base of the cone.

   (b) Find the slant height of the cone.



6. **[Paper 1 Style, Short Answer, Medium, 6 marks]**
A solid block of wax in the shape of a cylinder has a radius of 4 cm and a height of 10 cm. The wax is melted down and completely recast into identical solid spheres, each with a radius of 2 cm.

   (a) Calculate the volume of the original cylinder.

   (b) Calculate the volume of one of the solid spheres.

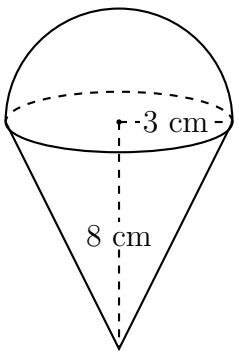
   (c) Hence, determine exactly how many complete spheres can be made.

7. **[Paper 2 Style, Longer Question, Medium, 6 marks]**
A dessert is made in the shape of a composite solid. It consists of a right circular cone with a hemisphere exactly on top of its flat base. Both the cone and the hemisphere share a radius of 3 cm. The vertical height of the conical part is 8 cm.

   (a) Calculate the volume of the hemispherical part.

   (b) Calculate the volume of the conical part.

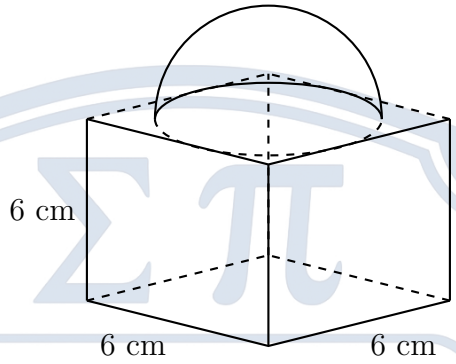
   (c) Calculate the total volume of the dessert.



8. **[Paper 2 Style, Longer Question, Hard, 7 marks]**
A solid glass paperweight consists of a hemisphere of diameter 6 cm sitting perfectly centred on top of a cuboid. The cuboid has a square base of length 6 cm and a height of 6 cm.

   (a) Calculate the total volume of the paperweight.

   (b) Calculate the total exposed surface area of the paperweight (including the flat bottom base of the cuboid).

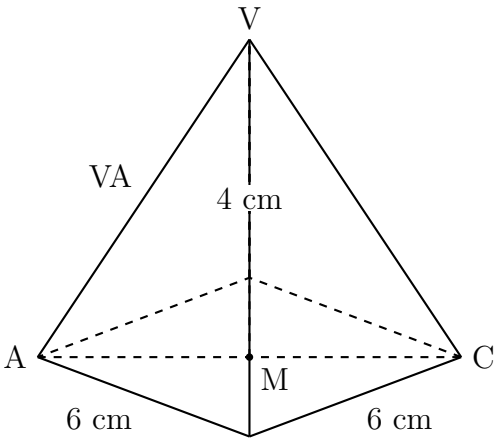


9. **[Paper 2 Style, Longer Question, Hard, 7 marks]**
A right solid pyramid VABCD has a square base ABCD with a side length of 6 cm. The vertical height of the pyramid, VM, is 4 cm, where M is the exact centre of the base.

   (a) Calculate the exact length of the base diagonal AC.

   (b) Calculate the length of the slant edge VA.

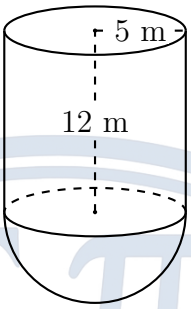
   (c) Calculate the angle that the slant edge VA makes with the flat base of the pyramid.



10. **[Paper 2 Style, Longer Question, Hard, 7 marks]**
A water storage tank is designed in the shape of a cylinder with a solid hemisphere perfectly attached to its bottom base. The tank has no top (it is open). The internal radius of the tank is 5 m and the internal height of the cylindrical section is 12 m.

    (a) Calculate the total internal capacity (volume) of the tank.

    (b) The inner surfaces of the tank are to be coated with a protective resin. Calculate the total internal surface area that needs to be coated.

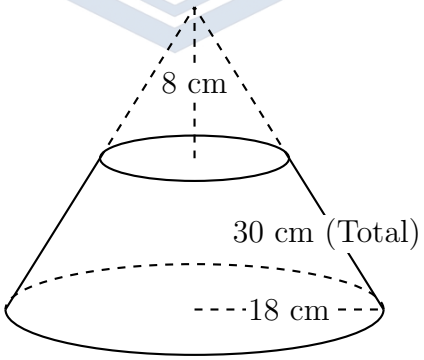


11. **[Paper 2 Style, Longer Question, Hard, 8 marks]**
A solid right circular cone has a base radius of 18 cm and a slant height of 30 cm. A smaller right circular cone, with a vertical height of 8 cm and a slant height of 10 cm, is removed from the top of the larger cone to create a frustum.

    (a) Calculate the radius of the base of the smaller cone which has been removed.

    (b) Calculate the curved surface area of the smaller removed cone.

    (c) Calculate the curved surface area of the remaining solid frustum.



12. **[Paper 1 Style, Short Answer, Hard, 5 marks]**
A weather balloon is in the shape of a perfect sphere. Initially, the balloon is filled with helium until its radius is 6 m.

   (a) Calculate the initial volume of the balloon.

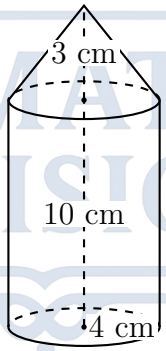
   (b) As it rises, the volume of the balloon expands by exactly 40%. Calculate the new radius of the balloon following this expansion.

13. **[Paper 2 Style, Longer Question, Very Hard, 8 marks]**
A composite 3D solid is formed by placing a solid right circular cone exactly on top of a solid cylinder. The cylinder has a radius of 4 cm and a height of 10 cm. The cone has the same base radius of 4 cm and a vertical height of 3 cm.

   (a) Calculate the total volume of the composite solid.

   (b) Find the slant height of the conical top.

   (c) Calculate the total exposed surface area of the composite solid (including the flat circular base).

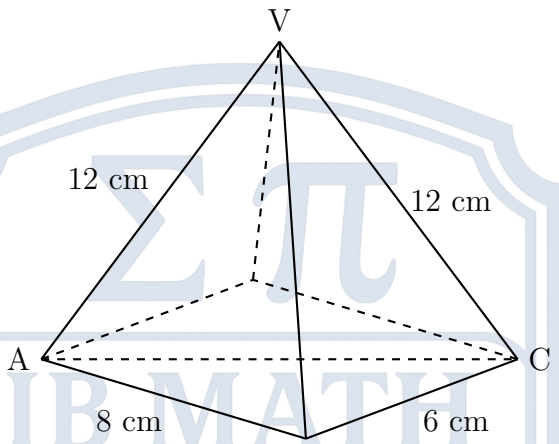


14. **[Paper 2 Style, Longer Question, Very Hard, 7 marks]**
A solid right rectangular-based pyramid VABCD has a base where $AB = 8$ cm and $BC = 6$ cm. The four slant edges VA, VB, VC, and VD are each 12 cm long. The apex of the pyramid is V.

(a) Calculate the exact length of the base diagonal AC.

(b) Calculate the angle $V\hat{C}A$ between the slant edge VC and the base diagonal AC.

(c) Calculate the vertical height of the pyramid from its apex V to the centre of the base.



15. **[Paper 2 Style, Longer Question, Very Hard, 5 marks]**
The volume of a solid hemisphere is given by V. The total surface area of this same hemisphere (including its flat circular base) is exactly 529 cm^2.

(a) Calculate the exact radius of the hemisphere, in terms of π.

(b) Hence, calculate the volume V of the hemisphere, giving your answer correct to 1 decimal place.