Unit 1: Exponentials and Logarithms
IB Math AA SL

Answer all questions. Show all working where appropriate. Use your graphic display calculator (GDC) to efficiently solve or verify equations where possible.

1. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
   Find the exact value of each of the following:

   (a) $\log_2 16$

   (b) $\log 25 + \log 4$

   (c) $\log_5 500 - \log_5 4$

2. **[Paper 1 Style, Short Answer, Easy, 6 marks]**
   Let $x = \ln 15$ and $y = \ln 3$. Write down the following expressions in terms of x and y:

   (a) $\ln 5$

   (b) $\ln 45$

   (c) $\ln 135$

3. **[Paper 2 Style, Short Answer, Easy, 6 marks]**
   Given that $2^m = 8$ and $2^n = 16$.

   (a) Write down the value of m and of n.

   (b) Hence, or otherwise, solve the equation $8^{2x+1} = 16^{2x-3}$.

4. **[Paper 2 Style, Longer Question, Medium, 6 marks]**
Let $f(x) = \ln(x + 2)$ for $x > -2$.

   (a) Find the exact coordinates of the x-intercept and the y-intercept.

   (b) State the equation of the vertical asymptote to the graph of f.

   (c) The graph of $y = f(x)$ intersects with its inverse, $y = f^{-1}(x)$, twice. Using your GDC, find the coordinates of these two points of intersection.

5. **[Paper 1 Style, Short Answer, Medium, 5 marks]**

   (a) Write the expression $3\ln 2 - \ln 4$ in the form $\ln k$, where $k \in \mathbb{Z}$.

   (b) Hence, or otherwise, solve $3\ln 2 - \ln 4 = -\ln x$.

6. **[Paper 2 Style, Short Answer, Medium, 6 marks]**
The number of bacteria, n, in a dish after t minutes is given by $n = 5231e^{0.12t}$.

   (a) Find the initial amount of bacteria.

   (b) Find the amount of bacteria after 12 minutes. Give your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

   (c) Find the exact value of t when $n = 2.7 \times 10^4$.

7. **[Paper 1 Style, Short Answer, Medium, 6 marks]**
Given that $\log_a 8 = 3$:

   (a) Find the value of a.

   (b) Find the value of $\log_a 64$.

   (c) Find the value of $\log_{a^2} 8$.

8. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
Solve the logarithmic equation $\log_6 3 + \log_6(2x) = 2 - \log_6 12$.

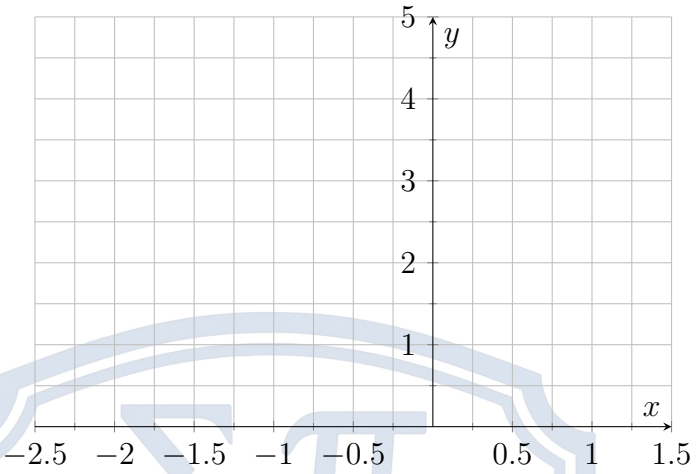
9. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
Find the exact value of each of the following:

   (a) $\log_4 32 + \log_4 8$

   (b) $64^{\log_4 3}$

10. **[Paper 2 Style, Longer Question, Hard, 8 marks]**
Let $f(x) = 0.5e^{2x} + 1$.

(a) On the grid below, sketch the graph of $y = f(x)$ for $-2 \leq x \leq 1$. Clearly label the y-intercept and draw the horizontal asymptote.



(b) The inverse of f can be written in the form $f^{-1}(x) = A\ln(b(x-c))$. Find the values of A, b, and c.

11. **[Paper 2 Style, Longer Question, Hard, 6 marks]**
Solve the equation $4^x - 3 \times 2^{x+2} = 64$.

12. **[Paper 1 Style, Longer Question, Hard, 7 marks]**
Solve the equation $\log_4(2-x) = \log_{16}(13-4x)$.

13. **[Paper 2 Style, Longer Question, Hard, 8 marks]**
A company's profits P (in thousands of dollars) in year y can be modelled by the function $P = P_0 y^k$, where P_0 and k are constants.

(a) Show that this equation can be written in the linear form $\log_{10} P = \log_{10} P_0 + k \log_{10} y$.

(b) A plot of $\log_{10} P$ on the vertical axis against $\log_{10} y$ on the horizontal axis produces a straight line. This line passes through the points $(0, 1.5)$ and $(2, 2.3)$. Find the value of k and the value of P_0.

14. **[Paper 2 Style, Extended Question, Very Hard, 8 marks]**
Solve the equation $2 \times 25^x - 30 \times 5^{x-1} = 1$. Give your answer to three significant figures.

15. **[Paper 1 Style, Extended Question, Very Hard, 7 marks]**
Solve the equation $16^x - 3(4^{x+1}) = 28$. Write your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers.