IB MATHEMATICS AA HL
AHL TOPIC 1 PRACTICE

Systems of Linear Equations (3x3) & Geometric Interpretations

Instructions to Candidates

- This practice paper contains **15** questions progressing from Easy to Very Hard.
- Each question indicates whether it is styled for **Paper 1 (No Calculator)** or **Paper 2 (Calculator Allowed)**.
- The paper tests syllabus topic AHL 1.16: Solving systems of linear equations with up to 3 variables, row reduction, and the geometric interpretation of intersecting planes.
- Answer all questions, showing all your working clearly.
- Total marks available: **81**.

Difficulty Progression

- **SECTION A (Easy):** Using technology to solve 3x3 systems directly, writing systems from word problems, and understanding basic geometric scenarios (unique point vs. no solution).
- **SECTION B (Medium):** Algebraic row reduction without a calculator, showing systems are dependent/inconsistent, and parameterising the intersection line of planes.
- **SECTION C (Hard):** Complex systems containing unknown constants (k, m, a, b). Using algebra or determinants to find specific conditions for unique solutions, infinite solutions, or no solutions.

SECTION A: EASY (Fundamentals)

Question 1 (3 Marks) — Paper 2 (Calculator Allowed)

Solve the following system of linear equations using your GDC:

$$\begin{aligned} 2x - 3y + z &= 10 \\ x + 4y - 2z &= -3 \\ -3x + y + 5z &= 7 \end{aligned}$$

Question 2 (2 Marks) — Paper 1 (No Calculator Allowed)

Three planes, Π_1, Π_2, and Π_3, are defined by a system of linear equations. When attempting to solve the system algebraically, a student arrives at the false statement $0 = 12$.
State two possible geometric interpretations of how these three planes intersect in 3D space.

Question 3 (4 Marks) — Paper 2 (Calculator Allowed)

A bakery sells three types of gift boxes containing cookies, brownies, and macarons:

- Box A contains 4 cookies, 2 brownies, and 3 macarons, and costs $25.
- Box B contains 5 cookies, 4 brownies, and 1 macaron, and costs $28.
- Box C contains 2 cookies, 5 brownies, and 4 macarons, and costs $31.

By setting up a system of linear equations, find the individual price of a single cookie, a single brownie, and a single macaron.

Question 4 (4 Marks) — Paper 1 (No Calculator Allowed)

Consider the following system of equations:

$$\begin{aligned} x + y + z &= 6 \\ x - y + z &= 2 \\ 2x + y - z &= 1 \end{aligned}$$

Use algebraic elimination to solve the system for x, y, and z.

Question 5 (2 Marks) — Paper 1 (No Calculator Allowed)

Given that a system of three linear equations has a unique solution $(x, y, z) = (4, -1, 3)$, fully describe the geometric arrangement of the three planes represented by the equations.

SECTION B: MEDIUM (Application & Algebraic Methods)

Question 6 (6 Marks) — Paper 1 (No Calculator Allowed)

Solve the following system of equations algebraically:

$$\begin{aligned} x + y + z &= 6 \\ 2x - y + 3z &= 9 \\ 3x + 2y - z &= 4 \end{aligned}$$

Question 7 (6 Marks) — Paper 1 (No Calculator Allowed)

Consider the planes defined by:

$$\begin{aligned} \Pi_1 : \quad x + y + z &= 1 \\ \Pi_2 : \quad 2x - y + z &= 2 \end{aligned}$$

Let the third plane be $\Pi_3 : \quad 3x + 2z = 3$.

(a) Show algebraically that this system of three equations has infinitely many solutions. **[3 marks]**

(b) Hence, express the infinite solutions in parametric form, using a parameter λ. **[3 marks]**

Question 8 (5 Marks) — Paper 2 (Calculator Allowed)

Determine if the following system is consistent or inconsistent. If it is consistent, find the solution. If it is inconsistent, explain the geometric reason why.

$$\begin{aligned} x + 2y - z &= 5 \\ 3x - y + 2z &= 8 \\ 4x + y + z &= 15 \end{aligned}$$

Question 9 (6 Marks) — Paper 1 (No Calculator Allowed)

Find the value of k for which the following system of equations does **not** have a unique solution:

$$\begin{aligned} x + 2y + 3z &= 4 \\ 2x + y - z &= 3 \\ 3x + 3y + kz &= 7 \end{aligned}$$

Question 10 (5 Marks) — Paper 2 (Calculator Allowed)

A system of equations can be written in matrix form as $MX = C$, where:

$$M = \begin{pmatrix} 2 & -1 & a \\ 1 & 1 & 2 \\ 4 & a & -1 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad C = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$$

Use the determinant of matrix M to find the values of a for which the system has a unique solution.

SECTION C: HARD / VERY HARD (Synthesis & Parameters)

Question 11 (8 Marks) — Paper 1 (No Calculator Allowed)

Consider the system of equations with real parameter k:

$$\begin{aligned} x + y + 2z &= 4 \\ 2x + 3y + 5z &= 7 \\ 3x + 4y + kz &= 12 \end{aligned}$$

(a) Use row reduction (elimination) to show that the system can be reduced to an equation of the form $(k-7)z = c$, finding the value of c. **[6 marks]**

(b) Hence, determine the value of k that makes the system inconsistent. **[2 marks]**

Question 12 (7 Marks) — Paper 2 (Calculator Allowed)

Three planes are given by the equations:

$$\begin{aligned} 2x + y + 3z &= 1 \\ -x + 2y + z &= p \\ 3x - y + 2z &= 4 \end{aligned}$$

Find the exact value of the parameter p such that the three planes intersect in a single common line.

Question 13 (6 Marks) — Paper 1 (No Calculator Allowed)

Three planes in 3D space are defined as:

$$\begin{aligned} \Pi_1 &: \quad x + 2y - 3z = 5 \\ \Pi_2 &: \quad -2x - 4y + 6z = k \\ \Pi_3 &: \quad x + y + z = 2 \end{aligned}$$

Given that Π_1 and Π_2 are strictly parallel (they do not coincide), state the possible values of k and describe the geometric shape formed by the intersection of the three planes.

Question 14 (8 Marks) — Paper 1 (No Calculator Allowed)

Consider the system of linear equations in x, y, z with real parameters a and b:

$$\begin{aligned} x - 2y + z &= 2 \\ 2x + y - 3z &= 5 \\ 4x - 3y - z &= a + b \\ x + 3y - 4z &= a - b \end{aligned}$$

Find the values of a and b required for this overdetermined system (4 equations, 3 unknowns) to be consistent. *(Hint: Notice that equation 3 and equation 4 are linear combinations of equation 1 and 2.)*

Question 15 (9 Marks) — Paper 1 (No Calculator Allowed)

[Full Parameter Analysis]
Consider the following system of linear equations containing the parameters m and k:

$$\begin{aligned} x + 2y + z &= 3 \\ 3x + y + 2z &= 4 \\ 4x + 3y + mz &= k \end{aligned}$$

Find the specific conditions on m and k such that the system has:

(a) Infinite solutions (planes intersect in a line). **[4 marks]**

(b) No solution (planes form a prism or are parallel). **[2 marks]**

(c) Exactly one unique solution (planes intersect at a point). **[3 marks]**