Unit 1: Simple Deductive Proofs
IB Math AA SL

Answer all questions. Show all working where appropriate. Use your graphic display calculator (GDC) to efficiently verify expansions or solve resulting equations where possible.

1. **[Paper 1 Style, Short Answer, Easy, 3 marks]**
   Prove the algebraic identity $(a-b)^2 - (a+b)^2 \equiv -4ab$.

2. **[Paper 1 Style, Short Answer, Easy, 3 marks]**
   Prove that $(4x-1)(2x+3) - (2x+1)^2 \equiv 2(2x-1)(x+2)$.

3. **[Paper 1 Style, Short Answer, Easy, 2 marks]**
   Prove that the square of any even integer is always a multiple of 4.

4. **[Paper 1 Style, Short Answer, Medium, 3 marks]**
   Prove that the sum of any three consecutive integers is always a multiple of 3.

5. **[Paper 2 Style, Short Answer, Medium, 4 marks]**
   Show that $\frac{1}{n+1} + \frac{1}{n^2+n} \equiv \frac{1}{n}$, where $n \in \mathbb{Z}^+$.

6. **[Paper 2 Style, Short Answer, Medium, 3 marks]**
   Prove algebraically that $x^2 - 3x + 3$ is positive for all real values of x.

7. **[Paper 1 Style, Short Answer, Medium, 4 marks]**

   (a) Show that $(3n+2)^2 - (n+2)^2 \equiv 8n^2 + 8n$, where $n \in \mathbb{Z}$.

   (b) Hence, or otherwise, prove that $(3n+2)^2 - (n+2)^2$ is a multiple of 8.

8. **[Paper 1 Style, Short Answer, Hard, 4 marks]**
Prove that the product of any two odd integers is always an odd integer.

9. **[Paper 2 Style, Longer Question, Hard, 4 marks]**
The sum of the squares of two consecutive integers is 313. Find the two possible pairs of integers that satisfy this condition.

10. **[Paper 1 Style, Longer Question, Hard, 5 marks]**

    (a) Factorise the quadratic expression $n^2 + 3n + 2$.

    (b) Hence show that $n^3 + 3n^2 + 2n \equiv n(n+1)(n+2)$.

    (c) Given that n is an even integer, deduce whether $n^3 + 3n^2 + 2n$ is odd or even. Justify your answer.