Topic: Worked Solutions & Mark Scheme
Applications of Functions to Real-Life Modelling

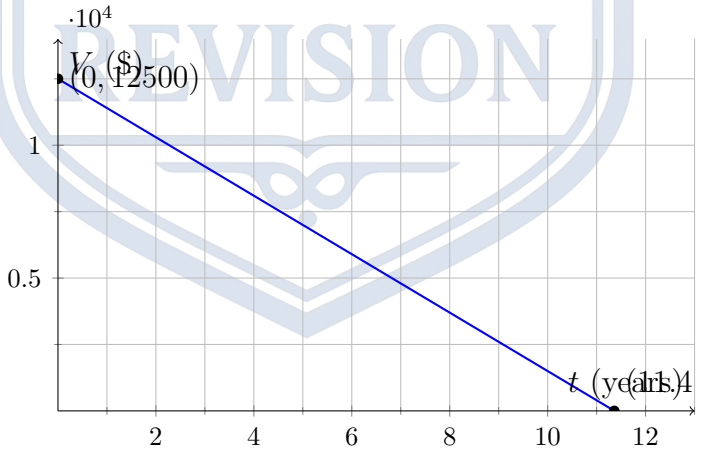
Marks are awarded for Method (M), Accuracy (A), and Reasoning (R). (M1) or (A1) indicates an implied mark.

1. **[Paper 1 Style, Short Answer, Easy, 5 marks]**

   (a) At $t = 0$, $V(0) = 12500 - 1100(0) = \$12\,500$. **A1**

   (b) $V(4) = 12500 - 1100(4) = 12500 - 4400 = \$8\,100$. **(M1)A1**

   (c) Set $V(t) = 0 \implies 0 = 12500 - 1100t$. **(M1)**
   $1100t = 12500 \implies t = \frac{12500}{1100} \approx 11.4$ years $(11.3636\ldots)$. **A1**

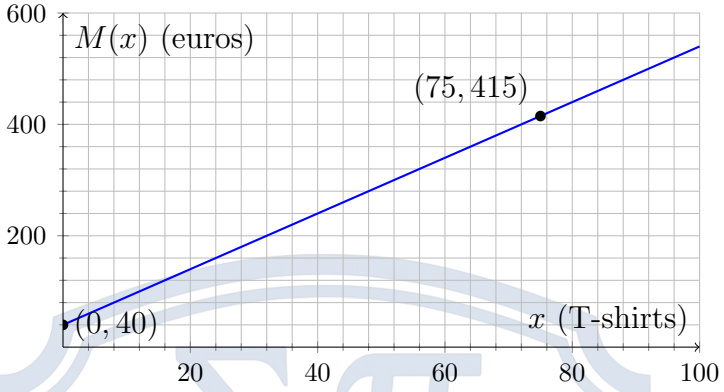


2. **[Paper 1 Style, Short Answer, Easy, 5 marks]**

(a) $M(10) = 5(10) + 40 = 50 + 40 = 90$ euros. **(M1)A1**

(b) Set $M(x) = 415 \implies 5x + 40 = 415$. **(M1)**
$5x = 375 \implies x = 75$ T-shirts. **A1**

(c) Sketch a straight line starting at $(0, 40)$ and passing through $(75, 415)$. **A1**

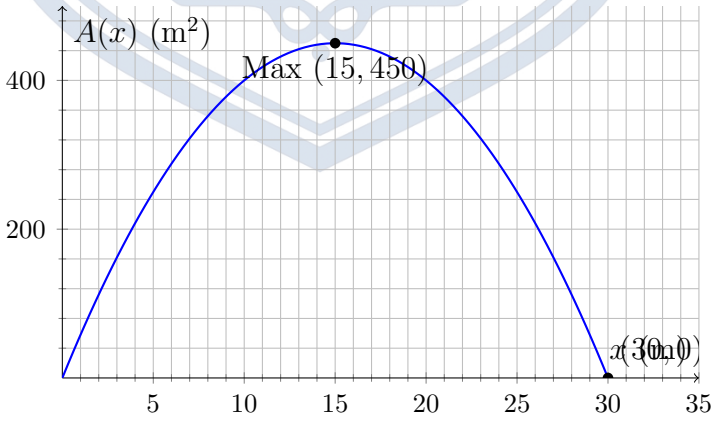


3. **[Paper 1 Style, Short Answer, Easy, 5 marks]**

(a) Set $A(x) = 0 \implies 2x(30 - x) = 0 \implies x = 0$ and $x = 30$. **(M1)A1**
Meaning: When width is 0 m or 30 m, the enclosure collapses and Area is 0. **R1**

(b) The vertex occurs exactly halfway between roots: $x = \frac{0+30}{2} = 15$ m. **(M1)**

(c) $A(15) = 60(15) - 2(15)^2 = 900 - 450 = 450$ m^2. **A1**

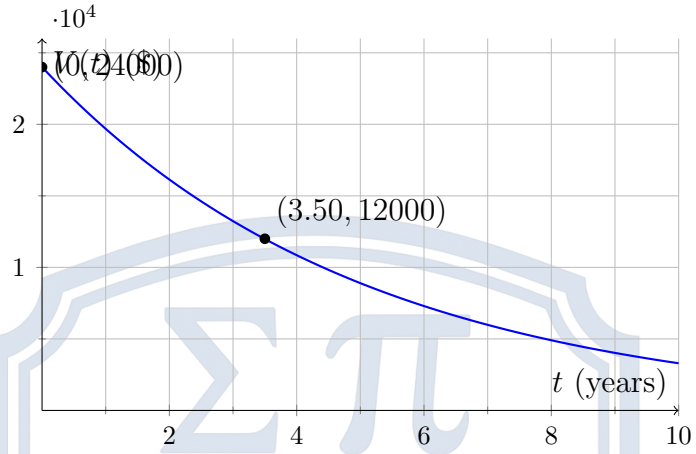


4. **[Paper 1 Style, Short Answer, Easy, 5 marks]**

(a) $t = 0 \implies V(0) = 24000(0.82)^0 = \$24\,000$. **A1**

(b) $V(5) = 24000(0.82)^5$. **(M1)**
$V(5) \approx 8897.66 \implies \$8\,898$. **A1**

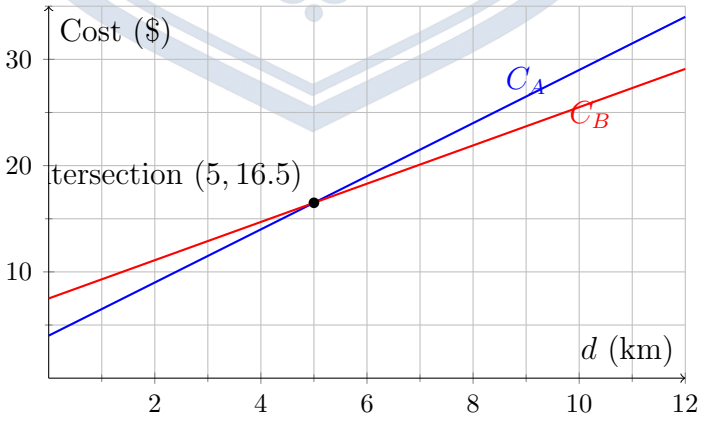
(c) Set $12000 = 24000(0.82)^t$. **CG50: Equation → Solver.** **(M1)**
$t = \frac{\ln(0.5)}{\ln(0.82)} \approx 3.50$ years $(3.496\ldots)$. **A1**



5. **[Paper 1 Style, Short Answer, Medium, 5 marks]**

(a) Correctly drawn C_A (starts at $y = 4$, gradient 2.5) and C_B (starts at $y = 7.5$, gradient 1.8). **A1A1**

(b) Set $C_A(d) = C_B(d) \implies 2.5d + 4.0 = 1.8d + 7.5$. **(M1)**
$0.7d = 3.5 \implies d = \frac{3.5}{0.7} = 5$ km. **A1A1**

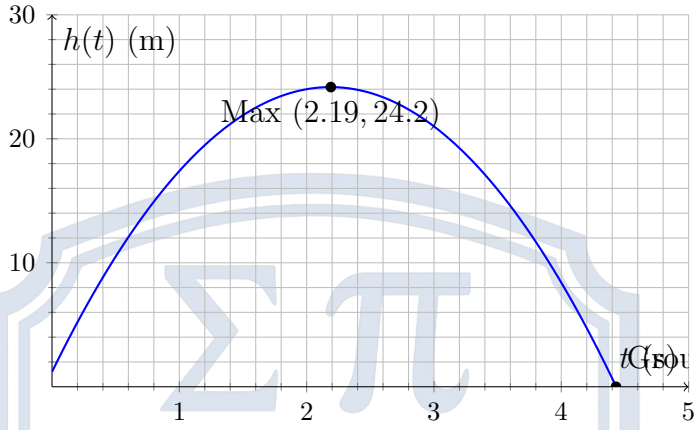


6. **[Paper 1 Style, Short Answer, Medium, 6 marks]**

(a) At $t = 0$, $h(0) = 1.2$ m. **A1**

(b) Max height at vertex: $t = \frac{-21}{2(-4.8)} \approx 2.19$ s. **(M1)**
$h(2.19) = -4.8(2.1875)^2 + 21(2.1875) + 1.2 \approx 24.2$ m $(24.168\ldots)$. **A1A1**

(c) Set $h(t) = 0 \implies -4.8t^2 + 21t + 1.2 = 0$. **CG50: Equation → Polynomial.** **(M1)**
$t = 4.43$ s $(4.431\ldots)$. (Ignore the negative time root). **A1**

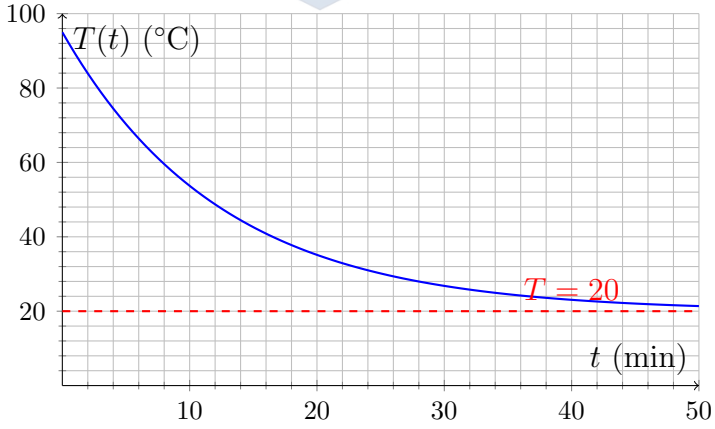


7. **[Paper 1 Style, Short Answer, Medium, 6 marks]**

(a) $t = 0 \implies T(0) = 20 + 75e^0 = 20 + 75 = 95°\text{C}$. **(M1)A1**

(b) $T(15) = 20 + 75e^{-0.08(15)} = 20 + 75e^{-1.2}$. **(M1)**
$T(15) \approx 42.6°\text{C}$ $(42.589\ldots)$. **A1**

(c) As $t \to \infty$, $e^{-0.08t} \to 0$. Asymptote: $T = 20$. **A1**
Meaning: 20°C represents the ambient room temperature that the coffee cools down to. **R1**

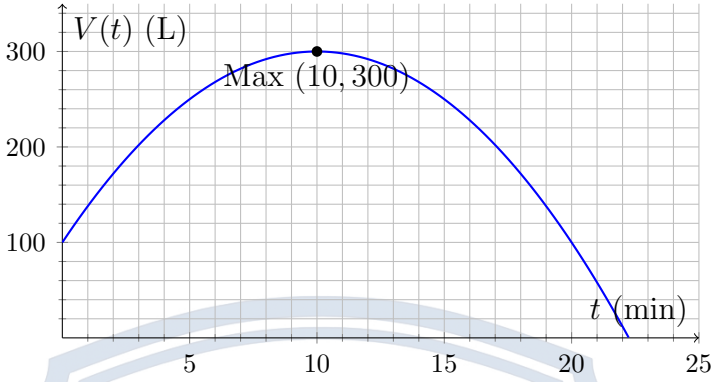


8. **[Paper 1 Style, Short Answer, Medium, 5 marks]**

(a) $t = 0 \implies V(0) = 100$ Litres. **A1**

(b) Max volume occurs at vertex $t = \frac{-b}{2a} = \frac{-40}{2(-2)} = 10$ min. **(M1)A1**

(c) $V(10) = -2(10)^2 + 40(10) + 100 = -200 + 400 + 100 = 300$ Litres. **(M1)A1**

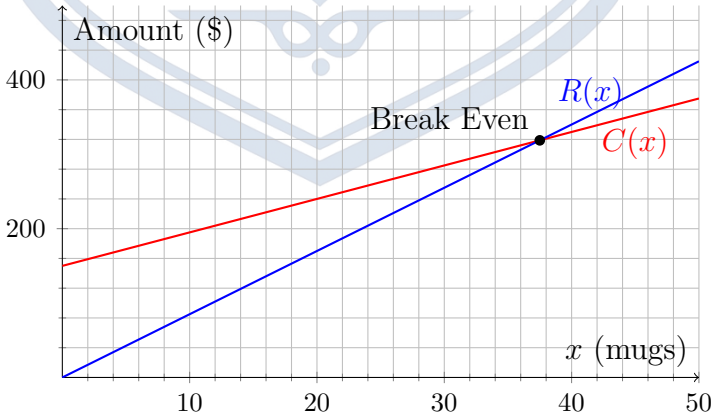


9. **[Paper 2 Style, Longer Question, Hard, 6 marks]**

(a) Sketch $C(x)$ starting at 150 and sloping gently. $R(x)$ starts at 0 and slopes steeper. **A1A1**

(b) Profit = Revenue − Cost. $P(x) = 8.50x - (150 + 4.50x) = 4.00x - 150$. **(M1)A1**

(c) Set $P(x) > 0 \implies 4.00x - 150 > 0 \implies 4.00x > 150 \implies x > 37.5$. **(M1)**
They must sell a minimum of 38 mugs to be strictly positive. **A1**

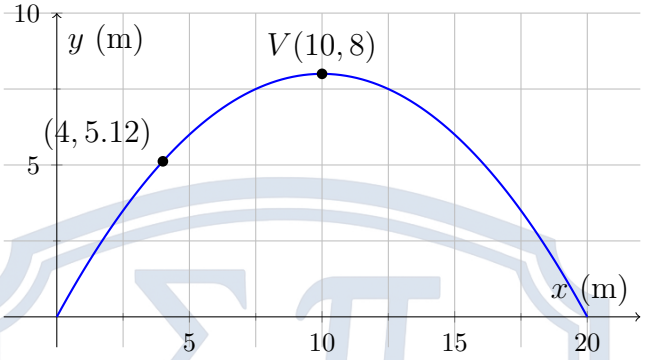


10. **[Paper 2 Style, Longer Question, Hard, 6 marks]**

(a) Vertex is halfway at 10 m across, height 8 m $\implies (10, 8)$. **A1**
Right base is at end of span $\implies (20, 0)$. **A1**

(b) $y = a(x-10)^2 + 8$. Substitute $(0,0)$: $0 = a(0-10)^2 + 8 \implies 100a = -8$. **(M1)**
$a = -0.08$. Equation: $y = -0.08(x-10)^2 + 8$. **A1**

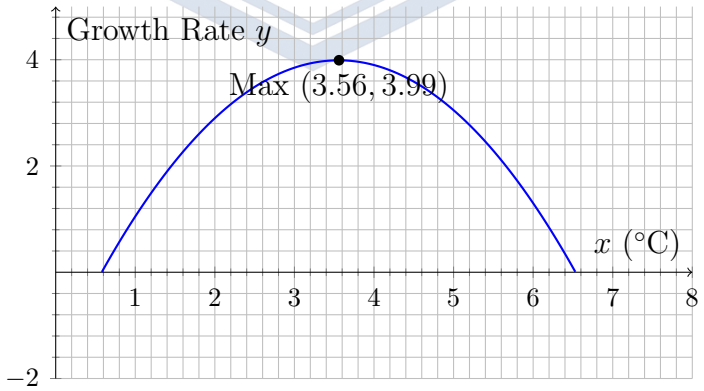
(c) $y(4) = -0.08(4-10)^2 + 8 = -0.08(36) + 8 = -2.88 + 8 = 5.12$ m. **(M1)A1**



11. **[Paper 1 Style, Short Answer, Hard, 6 marks]**

(a) **CG50: Graph → G-Solv → MAX**. (Or use $\frac{-b}{2a} = \frac{-3.2}{2(-0.45)} \approx 3.56$). **(M1)**
$x \approx 3.56$°C $(3.555\ldots)$. **A1**
Max growth rate $y \approx 3.99$ units $(3.988\ldots)$. **A1**

(b) **CG50: Equation → Polynomial → Degree 2** (or **G-Solv → ROOT**). **(M1)**
$x_1 \approx 0.58$°C $(0.584\ldots)$. **A1**
$x_2 \approx 6.53$°C $(6.526\ldots)$. **A1**



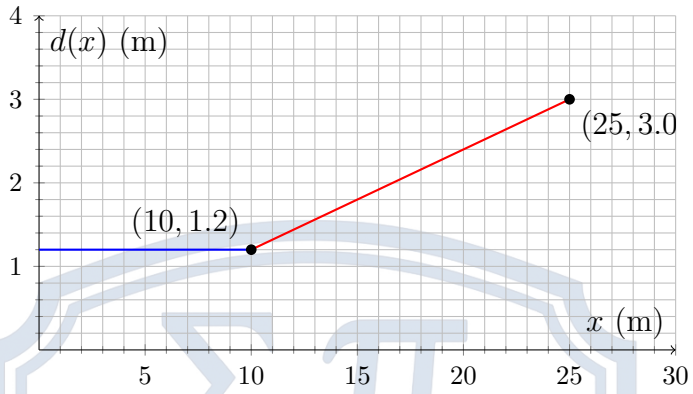
12. **[Paper 2 Style, Longer Question, Hard, 6 marks]**

(a) Flat line from $(0, 1.2)$ to $(10, 1.2)$, then slanted line down to $(25, 3.0)$. **A1A1**

(b) Gradient $m = \frac{3.0-1.2}{25-10} = \frac{1.8}{15} = 0.12$. **(M1)A1**

(c) $d(x) - 1.2 = 0.12(x - 10) \implies d(x) = 0.12x - 1.2 + 1.2 \implies d(x) = 0.12x$. **(M1)A1**

Note: The depth function happens to intersect the origin if projected backward.



13. **[Paper 2 Style, Longer Question, Very Hard, 6 marks]**

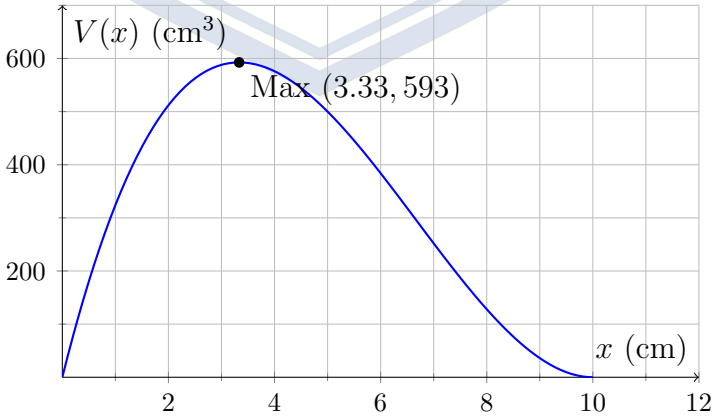
(a) Cutout size x must be positive, and we cannot cut out more than half the side length. **(M1)**

Domain: $0 < x < 10$. **A1A1**

(b) **CG50: Graph → G-Solv → MAX** on $Y1 = x(20 - 2x)^2$. **(M1)**

Max occurs at $x = 3.33$ cm $(3.333\ldots)$. **A1**

(c) Max Volume $V(3.33) \approx 593$ cm^3 $(592.59\ldots)$. **A1**

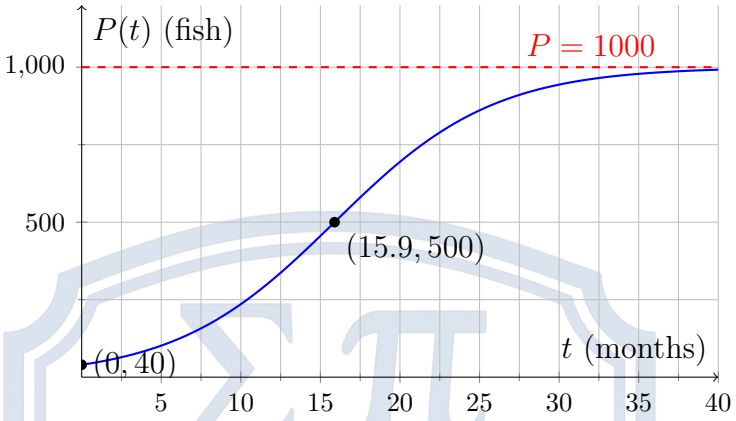


14. **[Paper 2 Style, Longer Question, Very Hard, 6 marks]**

(a) $t = 0 \implies P(0) = \frac{1000}{1+24e^0} = \frac{1000}{25} = 40$ fish. **(M1)A1**

(b) As $t \to \infty$, $e^{-0.2t} \to 0$, so $P(t) \to \frac{1000}{1+0} = 1000$. Asymptote: $P = 1000$. **A1**
Meaning: The lake has a maximum carrying capacity of 1000 fish. **R1**

(c) Set $500 = \frac{1000}{1+24e^{-0.2t}}$. **CG50: Equation → Solver**. **(M1)**
$t = \frac{\ln(24)}{0.2} \approx 15.9$ months $(15.890\ldots)$. **A1**



15. **[Paper 2 Style, Longer Question, Very Hard, 6 marks]**

(a) Sketch $V_A(t)$ decaying from 20 downwards, and $V_B(t)$ growing from 10 upwards. **A1A1**

(b) Set $20(0.8)^t = 10(1.05)^t$. **CG50: Graph → G-Solv → ISCT**. **(M1)**
$t \approx 2.55$ years $(2.5539\ldots)$. **A1A1**

(c) The value at intersection is $V \approx \$11.3$k or \$11 328 $(11.328\ldots)$. **A1**