

IB MATHEMATICS AA HL

AHL TOPIC 5 PRACTICE

Substitution, By Parts, Partial Fractions & Volumes of Revolution

Instructions to Candidates

- This practice paper contains **20** questions progressing from Easy to Very Hard.
- Each question indicates whether it is styled for **Paper 1 (No Calculator)** or **Paper 2 (Calculator Allowed)**.
- The paper tests syllabus topics AHL 5.15, 5.16 and 5.17: Integrals of a^x and inverse trig, partial fractions, integration by substitution, integration by parts, areas enclosed by the y -axis, and volumes of revolution about both the x and y axes.
- Answer all questions, showing all your working clearly.
- Total marks available: **108**.

Difficulty Progression

- **SECTION A (Easy):** Standard integrals for a^x and $\arcsin x$, basic integration by parts (xe^x), simple given substitutions, and fundamental volumes of revolution.
- **SECTION B (Medium):** Evaluating integrals using partial fractions, logarithmic integration by parts, areas mapped to the y -axis, and volumes between two curves.
- **SECTION C (Hard):** Repeated and looping integration by parts ($e^x \sin x$), complex trigonometric substitutions, formal volume derivations, and finding unknown constants from generated volumes.

SECTION A: EASY (Fundamentals)**Question 1 (3 Marks) — Paper 1 (No Calculator Allowed)**

Find the indefinite integral:

$$\int 5^x dx$$

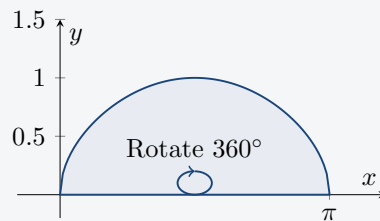
Question 2 (3 Marks) — Paper 1 (No Calculator Allowed)

Evaluate the following integral, yielding an inverse trigonometric function:

$$\int \frac{1}{\sqrt{1-9x^2}} dx$$

Question 3 (4 Marks) — Paper 1 (No Calculator Allowed)Use the substitution $u = x^2 + 1$ to find the exact value of the indefinite integral:

$$\int 2x(x^2 + 1)^4 dx$$

Question 4 (4 Marks) — Paper 2 (Calculator Allowed)The region bounded by the curve $y = \sqrt{\sin x}$ and the x -axis, for $0 \leq x \leq \pi$, is rotated 360° (2π radians) about the x -axis to form a solid of revolution.

Calculate the exact volume of this solid.

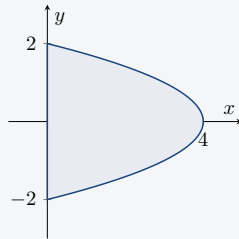
Question 5 (4 Marks) — Paper 1 (No Calculator Allowed)

Use the method of integration by parts to find:

$$\int xe^{3x} dx$$

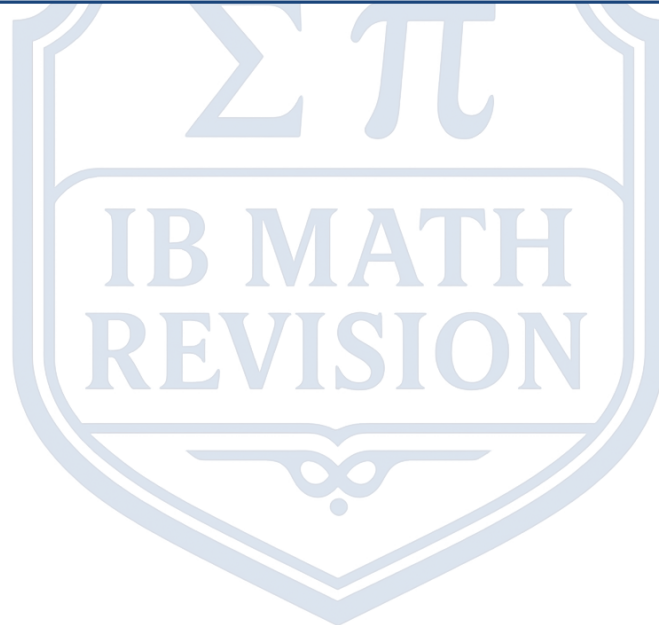
Question 6 (5 Marks) — Paper 1 (No Calculator Allowed)

Find the exact area of the region completely enclosed by the curve $x = 4 - y^2$ and the y -axis.

**Question 7 (5 Marks) — Paper 1 (No Calculator Allowed)**

Express the algebraic fraction $\frac{3}{(x-2)(x+1)}$ in partial fractions. Hence, find:

$$\int \frac{3}{x^2 - x - 2} dx$$



SECTION B: MEDIUM (Application & Algebraic Methods)

Question 8 (5 Marks) — Paper 1 (No Calculator Allowed)

Use the substitution $u = e^x$ to find the exact value of the definite integral:

$$\int_0^{\ln 2} \frac{e^x}{e^{2x} + 1} dx$$

Question 9 (5 Marks) — Paper 1 (No Calculator Allowed)

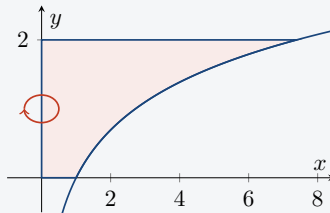
Using integration by parts, evaluate:

$$\int \ln x dx$$

(Hint: Let $u = \ln x$ and $\frac{dv}{dx} = 1$).

Question 10 (6 Marks) — Paper 2 (Calculator Allowed)

The region enclosed by the curve $y = \ln x$, the x -axis, the y -axis, and the horizontal line $y = 2$ is rotated 360° about the y -axis.



Find the exact volume of the solid generated.

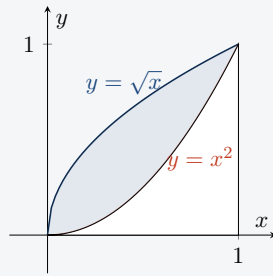
Question 11 (6 Marks) — Paper 1 (No Calculator Allowed)

Use a suitable trigonometric substitution to evaluate exactly:

$$\int_0^{\pi/2} \cos x \sin^3 x dx$$

Question 12 (6 Marks) — Paper 2 (Calculator Allowed)

The region bounded by the intersecting curves $y = x^2$ and $y = \sqrt{x}$ is rotated 360° around the x -axis.



Calculate the exact volume of the solid of revolution formed.

Question 13 (6 Marks) — Paper 1 (No Calculator Allowed)

Use integration by parts to find the indefinite integral:

$$\int \arcsin x \, dx$$

Question 14 (6 Marks) — Paper 1 (No Calculator Allowed)

Evaluate the following integral by first splitting the fraction algebraically into two parts:

$$\int \frac{x + 3}{x^2 + 4x + 5} \, dx$$

(Hint: Notice that $x^2 + 4x + 5 = (x + 2)^2 + 1$).

SECTION C: HARD / VERY HARD (Synthesis & Proof)

Question 15 (7 Marks) — Paper 1 (No Calculator Allowed)

Use **repeated** integration by parts to evaluate the definite integral exactly:

$$\int_0^1 x^2 e^{-x} dx$$

Question 16 (7 Marks) — Paper 1 (No Calculator Allowed)

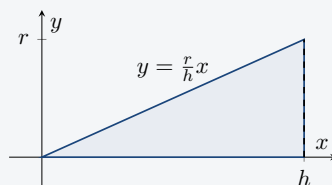
Let $I = \int e^x \cos x dx$. By applying integration by parts twice, show that the integral "loops" back to its original form, and hence formally prove that:

$$\int e^x \cos x dx = \frac{1}{2} e^x (\sin x + \cos x) + C$$

Question 17 (7 Marks) — Paper 1 (No Calculator Allowed)

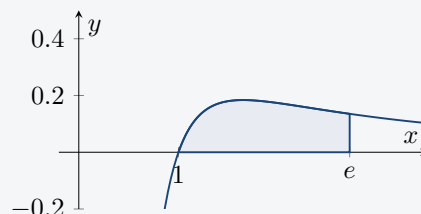
By rotating the straight line segment joining the origin $(0, 0)$ to the point (h, r) around the x -axis, formally prove that the volume of a right circular cone is given by:

$$V = \frac{1}{3} \pi r^2 h$$



Question 18 (7 Marks) — Paper 2 (Calculator Allowed)

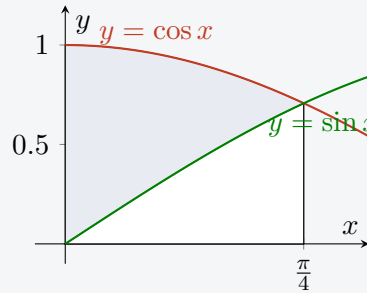
Consider the curve $f(x) = \frac{\ln x}{x^2}$ for $x > 0$.



Find the exact area enclosed by the curve, the x -axis, and the vertical line $x = e$.

Question 19 (7 Marks) — Paper 2 (Calculator Allowed)

The region bounded by the y -axis, and the curves $y = \sin x$ and $y = \cos x$ in the interval $0 \leq x \leq \frac{\pi}{4}$ is rotated 360° around the x -axis.



Find the exact volume of the solid generated. (*Hint: You may find the double angle identity $\cos(2x) = \cos^2 x - \sin^2 x$ useful.*)

Question 20 (7 Marks) — Paper 2 (Calculator Allowed)

Find the exact value of the positive constant k such that the area between the curve $y = \frac{k}{x^2+1}$, the x -axis, and the vertical lines $x = -1$ and $x = \sqrt{3}$ is exactly equal to π .

IB MATH
REVISION