Topic 1: Sequences and Series
IB Math AI SL

Answer all questions. Show all working where appropriate. Solutions found from a Graphic Display Calculator (GDC) should be supported by suitable working. Total: 112 marks.

1. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
An arithmetic sequence has the following first three terms: $4, 9, 14, \ldots$

   (a) Find the common difference, d.

   (b) Find the value of the 15th term, u_{15}.

   (c) Find the sum of the first 15 terms of the sequence, S_{15}.

2. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
A geometric sequence is defined by the terms: $3, 12, 48, 192, \ldots$

   (a) Write down the common ratio, r.

   (b) Find the exact value of the 8th term, u_8.

   (c) Find the sum of the first 8 terms of the sequence, S_8.

3. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
The nth term of a sequence is given by the formula $u_n = 5n - 8$.

   (a) Find the value of u_{10}.

   (b) Write down an expression for $u_{n+1} - u_n$ and simplify it.

   (c) Hence, explain why the sequence is arithmetic.

4. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
The table shows the first four terms of three sequences: u_n, v_n, and w_n.

n	1	2	3	4
u_n	12	24	48	72
v_n	12	24	36	48
w_n	12	24	48	96

(a) State which sequence is arithmetic and find its common difference.

(b) State which sequence is geometric and find its common ratio.

5. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
The first term of an infinite geometric sequence is 4. The sum of the infinite sequence is 200.

(a) Find the common ratio, r.

(b) Find the sum of the first 8 terms, S_8, correct to one decimal place.

6. **[Paper 1 Style, Short Answer, Medium, 3 marks]**
Using the summation tool on your Graphic Display Calculator, evaluate the following arithmetic series:

$$\sum_{k=1}^{12}(5k-2)$$

Show the values you entered into your calculator.

7. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
The fourth term of an arithmetic sequence is 17 and the tenth term is 41.

(a) Using your GDC's equation solver or otherwise, find the common difference, d, and the first term, u_1.

(b) Calculate the sum of the first 20 terms, S_{20}.

8. **[Paper 1 Style, Short Answer, Medium, 4 marks]**
The second term of a geometric sequence is -6 and the fifth term is 162.

(a) Find the common ratio, r.

(b) Find the first term, u_1.

9. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
A local theater is designed with 15 rows of seats. The first row has 20 seats, the second row has 22 seats, the third row has 24 seats, and this pattern continues.

   (a) Find the number of seats in the final (15th) row.

   (b) Calculate the total seating capacity of the theater.

10. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
Mia baked a very large apple pie that she cuts into slices to share with her friends. The smallest slice is cut first. The volume of each successive slice of pie forms a geometric sequence. The second smallest slice has a volume of 30 cm^3. The fifth smallest slice has a volume of 240 cm^3.

   (a) Find the common ratio of the sequence.

   (b) Find the volume of the smallest slice of pie.

   (c) The total apple pie has a volume of 61 425 cm^3. Find the total number of slices Mia can cut from this pie.

11. **[Paper 1 Style, Short Answer, Hard, 4 marks]**
How many terms of the arithmetic series $11 + 16 + 21 + 26 + \ldots$ are needed to strictly exceed a sum of 450? Use your GDC to solve.

12. **[Paper 2 Style, Longer Question, Hard, 7 marks]**
A mining company extracts 500 tons of iron ore in their first month of operation. However, due to depleting resources, their production drops by exactly 15 tons each successive month.

   (a) Write down the expected production in the 12th month.

   (b) Find the total amount of iron ore extracted over the first 24 months.

   (c) Determine during which month the production will drop to zero.

13. **[Paper 2 Style, Longer Question, Hard, 6 marks]**
A rubber ball is dropped from a height of 10 m onto a hard surface. After each bounce, it returns to 80% of its previous height.

   (a) Find the height the ball reaches after the 4th bounce.

   (b) Find the total theoretical distance travelled by the ball (up and down) until it completely stops bouncing.

14. **[Paper 1 Style, Short Answer, Hard, 6 marks]**
Consider the sum defined by $S = \sum_{k=1}^{10} \ln(2^k)$.

(a) Write out the first three terms of this series and show algebraically that it forms an arithmetic sequence.

(b) Express the total sum S in the form $a \ln 2$, where $a \in \mathbb{Z}^+$.

15. **[Paper 1 Style, Short Answer, Hard, 5 marks]**
Find the first term of the geometric sequence $6, 6\sqrt{2}, 12, 12\sqrt{2}, \ldots$ which strictly exceeds the value of 1400.

16. **[Paper 2 Style, Longer Question, Hard, 7 marks]**
Consider an arithmetic sequence with first term $u_1 = 2$ and a non-zero common difference d. It is known that the 1st, 3rd, and 7th terms of this arithmetic sequence form three consecutive terms of a geometric sequence.

(a) Express the 3rd and 7th terms of the arithmetic sequence in terms of d.

(b) Show that $(2 + 2d)^2 = 2(2 + 6d)$ and hence find the value of d.

(c) Find the common ratio, r, of the geometric sequence.

17. **[Paper 2 Style, Longer Question, Very Hard, 8 marks]**
Charlie and Daniella each began a fitness programme. On day one, they both ran 500 m. On each subsequent day, Charlie ran 100 m more than the previous day whereas Daniella increased her distance by 2% of the distance she ran on the previous day.

(a) Calculate how far Charlie ran on day 20.

(b) Calculate how far Daniella ran on day 20.

(c) On day n of the fitness programme, Daniella runs further than Charlie in a single day for the first time. Using your GDC, find the value of n.

18. **[Paper 2 Style, Longer Question, Very Hard, 10 marks]**
The sum of the first n terms of an arithmetic sequence, S_n, is given by $S_n = 3n^2 - 2n$.

(a) Find the values of S_1 and S_2.

(b) Hence, find the values of the first term u_1, and the second term u_2.

(c) Find d, the common difference of the sequence.

(d) Find u_{10}, the tenth term of the sequence.

(e) Find the greatest value of n, for which the individual term u_n is strictly less than 100.

19. **[Paper 2 Style, Longer Question, Very Hard, 8 marks]**
An infinite geometric series has first term $u_1 = a$ and second term $u_2 = \frac{1}{4}a^2 - 3a$, where $a > 0$.

(a) Find the common ratio r in terms of a.

(b) Find the range of possible values for a such that the sum to infinity of this geometric series exists.

20. **[Paper 2 Style, Longer Question, Very Hard, 7 marks]**
The first terms of an infinite geometric sequence, u_n, are $2, 6, 18, 54, \ldots$
The first terms of a second infinite geometric sequence, v_n, are $2, -6, 18, -54, \ldots$
The terms of a third sequence, w_n, are defined by the addition of the two sequences: $w_n = u_n + v_n$.

(a) Write down the first three **non-zero** terms of the sequence w_n.

(b) Using your GDC, evaluate the sum of the first 10 **non-zero** terms of the sequence w_n.