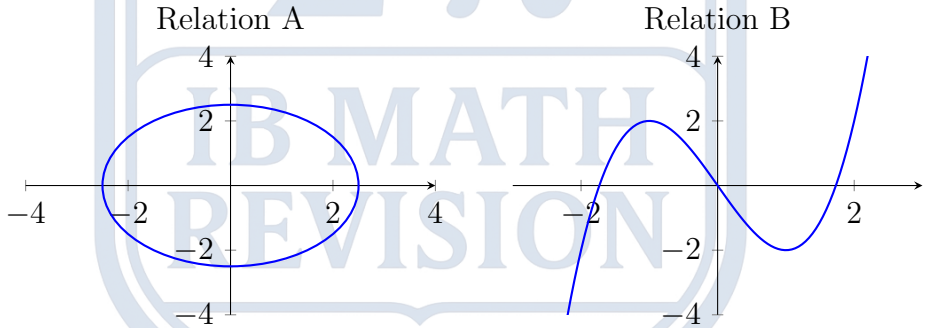
Topic 2: Functions, Domain, and Range
IB Math AI SL

Answer all questions. Show all working where appropriate. Total: 115 marks.

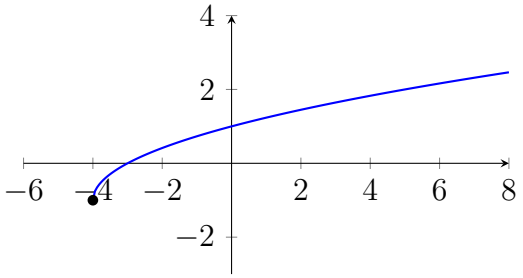
1. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
Determine which of the two relations graphed below represents a function. Justify your answer using the vertical line test. State the domain and range for Relation B.



2. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
Consider the radical function $f(x) = \sqrt{x+4} - 1$. The graph of the function is shown below.

   (a) State the domain of $f(x)$.

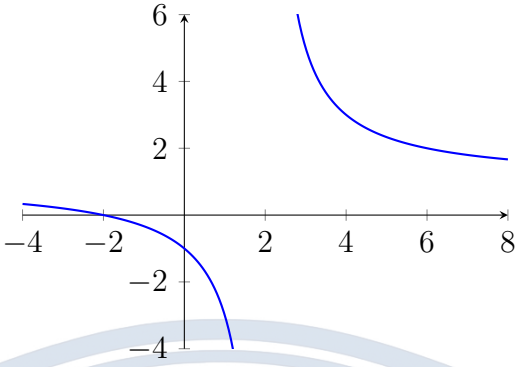
   (b) State the range of $f(x)$.



3. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
The graph of the rational function $g(x) = \frac{x+2}{x-2}$ is shown below.

(a) Write down the equations of the vertical and horizontal asymptotes.

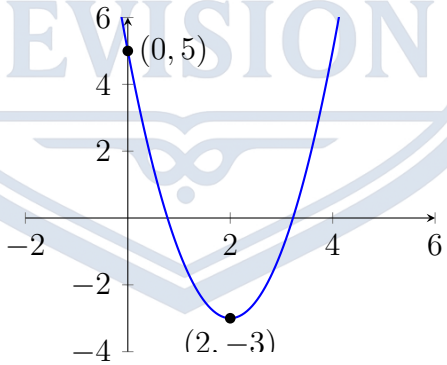
(b) Hence, state the domain and range of $g(x)$.



4. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
The graph of a quadratic function $y = f(x)$ is shown below. It has a vertex at $V(2, -3)$ and passes through $(0, 5)$.

(a) State the domain of the function.

(b) State the range of the function.



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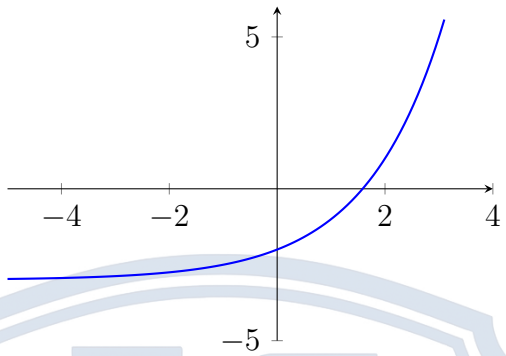
5. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
Consider the exponential function $h(x) = 2^x - 3$. The graph of the function is shown below.

   (a) Write down the domain of $h(x)$.

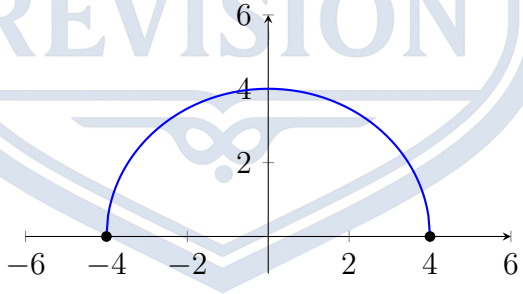
   (b) Determine the equation of the horizontal asymptote and hence state the range of $h(x)$.



6. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
The graph of the semi-circle $f(x) = \sqrt{16 - x^2}$ is plotted below.

   (a) State the maximal domain of $f(x)$.

   (b) State the range of $f(x)$.



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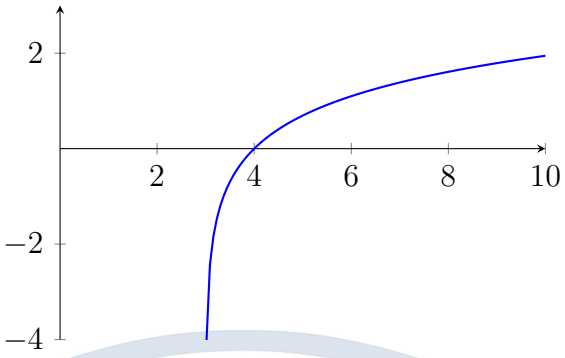
7. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
Consider the natural logarithmic function $p(x) = \ln(x - 3)$.

   (a) Write down the equation of the vertical asymptote.

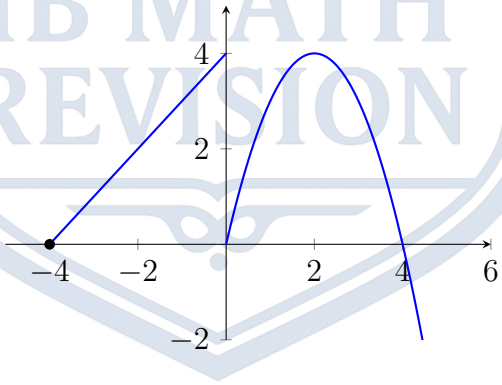
   (b) State the domain and range of $p(x)$.



8. **[Paper 1 Style, Short Answer, Medium, 6 marks]**
A piecewise function $f(x)$ is graphed below.

   (a) Determine the domain of $f(x)$.

   (b) Determine the range of $f(x)$.



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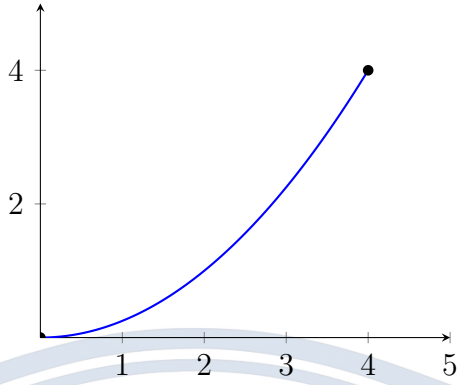
9. **[Paper 1 Style, Short Answer, Hard, 6 marks]**
The graph of a one-to-one function $y = f(x)$ is shown below.

   (a) State the domain and range of $f(x)$.

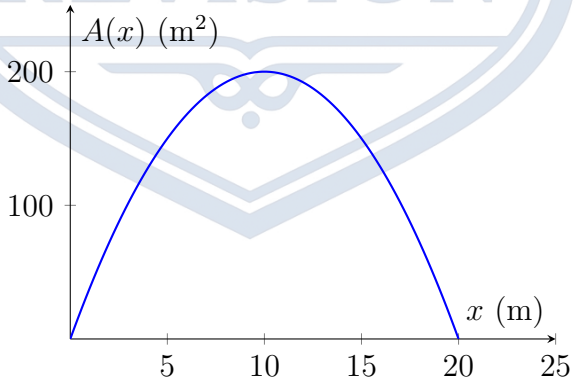
   (b) Hence, state the domain and range of its inverse function, $f^{-1}(x)$.



10. **[Paper 2 Style, Longer Question, Hard, 7 marks]**
A farmer uses 40 metres of fencing to form a rectangular enclosure against a straight wall (no fence needed along the wall). The width is x metres. The area is given by $A(x) = 40x - 2x^2$. The graph is shown below.

   (a) State the contextual domain for x (the possible widths of the enclosure).

   (b) Using the graph or otherwise, find the maximum area and state the contextual range of $A(x)$.



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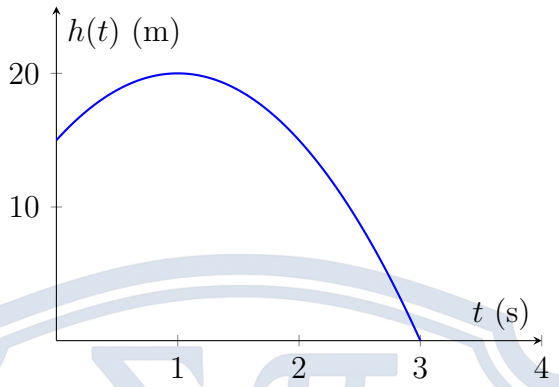
11. **[Paper 2 Style, Longer Question, Hard, 6 marks]**
A ball is thrown upwards from a balcony. Its height $h(t)$ in metres after t seconds is modelled by $h(t) = -5t^2 + 10t + 15$. The flight path is graphed below.

(a) Find the exact time t when the ball hits the ground, and state the contextual domain.

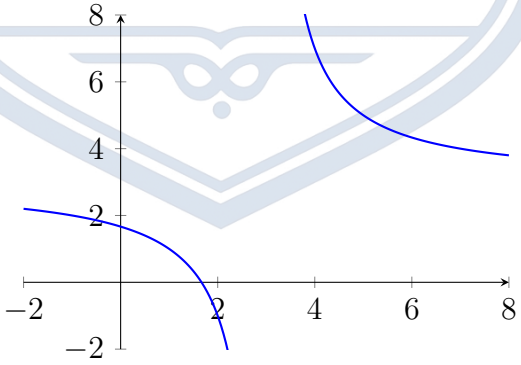
(b) Find the maximum height reached, and state the contextual range.



12. **[Paper 1 Style, Short Answer, Hard, 6 marks]**
Consider the rational function $f(x) = \frac{3x-5}{x-3}$. Its graph is shown below.

(a) State the domain and range of $f(x)$.

(b) By considering the domain and range, explain why the inverse function $f^{-1}(x)$ has the exact same domain and range as $f(x)$.



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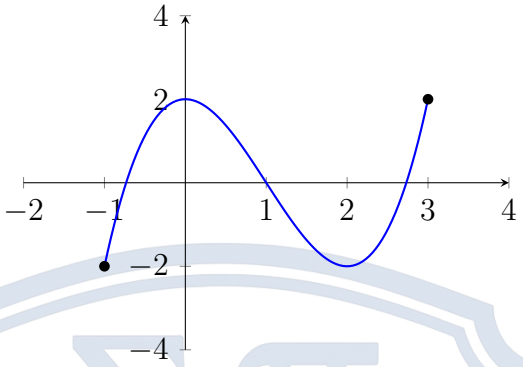
13. **[Paper 2 Style, Longer Question, Hard, 6 marks]**
The cubic function $f(x) = x^3 - 3x^2 + 2$ is plotted on the restricted domain $-1 \leq x \leq 3$.

(a) Identify the coordinates of the local minimum and local maximum from the graph.

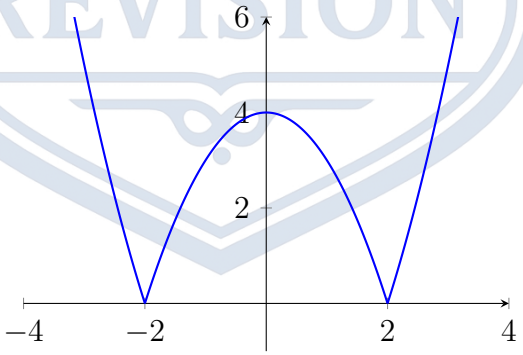
(b) Hence, determine the exact range of $f(x)$ on this restricted domain.



14. **[Paper 1 Style, Short Answer, Very Hard, 6 marks]**
The absolute value function $g(x) = |x^2 - 4|$ is graphed below.

(a) Write down the domain of $g(x)$.

(b) Determine the range of $g(x)$, justifying your answer algebraically.



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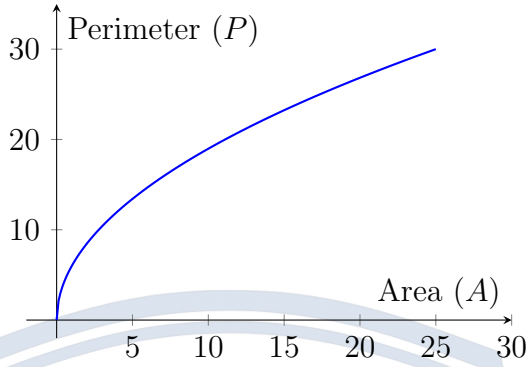
15. **[Paper 2 Style, Longer Question, Very Hard, 6 marks]**
The relationship between the perimeter P and Area A of a specific family of shapes is given by $P = 6\sqrt{A}$.

(a) State the domain of this mathematical model in context.

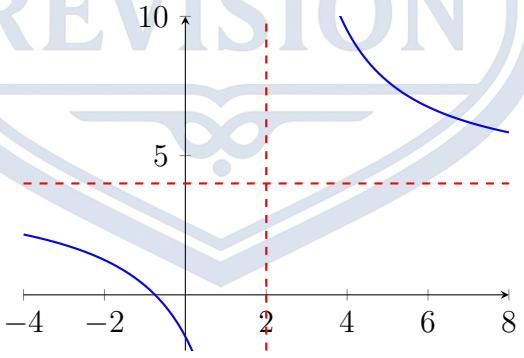
(b) State the range of this mathematical model in context.



16. **[Paper 1 Style, Short Answer, Very Hard, 6 marks]**
A rational function $f(x) = \frac{ax+3}{x-b}$ is shown below. It has a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = 4$.

(a) Determine the exact values of a and b.

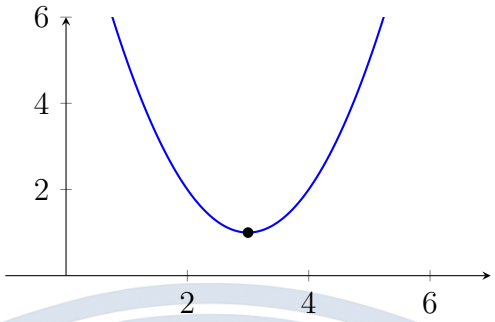
(b) Hence, state the domain and range of the inverse function $f^{-1}(x)$.



17. **[Paper 2 Style, Longer Question, Very Hard, 6 marks]**
The quadratic function $f(x) = (x-3)^2 + 1$ is graphed below. The domain must be restricted to $x \geq k$ so that the inverse function $f^{-1}(x)$ exists.

(a) Find the minimum value of k.

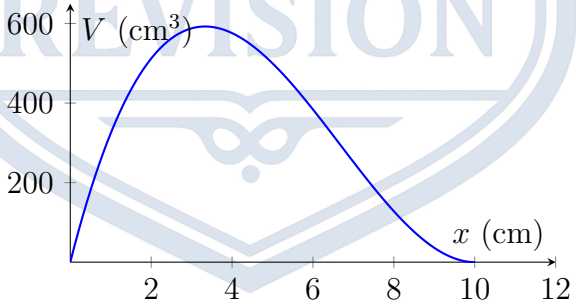
(b) For this restricted domain, state the domain and range of $f^{-1}(x)$.



18. **[Paper 2 Style, Longer Question, Very Hard, 6 marks]**
A manufacturer creates an open box from a 20 cm by 20 cm piece of cardboard by cutting squares of side x from each corner and folding up the sides. The volume is $V(x) = x(20-2x)^2$.

(a) State the contextual domain for x.

(b) Using your GDC to find the maximum volume, state the range for $V(x)$.



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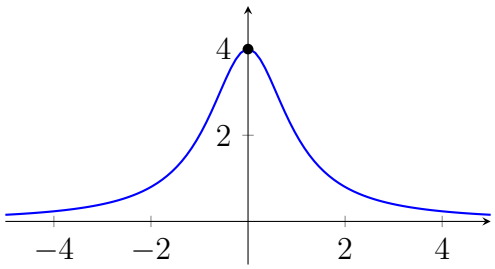
19. **[Paper 1 Style, Short Answer, Very Hard, 6 marks]**
Consider the function $f(x) = \frac{4}{x^2+1}$. Its graph is shown below.

(a) State the domain of $f(x)$.

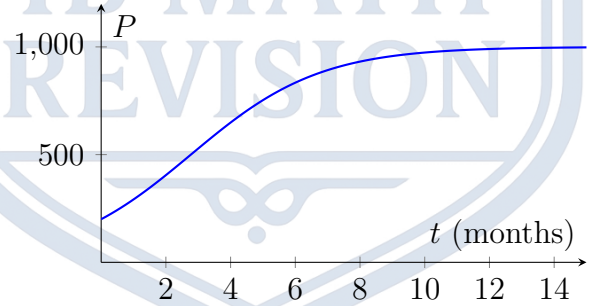
(b) By considering the maximum point and the horizontal asymptote, state the range of $f(x)$.



20. **[Paper 2 Style, Longer Question, Very Hard, 6 marks]**
A population of fish in a lake is modelled by the logistic function $P(t) = \frac{1000}{1+4e^{-0.5t}}$, where $t \geq 0$ is the time in months. The graph is shown below.

(a) Find the initial population $P(0)$.

(b) State the equation of the horizontal asymptote and hence write down the range of $P(t)$ for $t \geq 0$.



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