Unit 4: Probability Concepts
IB Math AA SL

Answer all 15 questions. Show all working. For Paper 1 questions, use analytical fraction methods. For Paper 2 questions, use your graphic display calculator (GDC) efficiently.

1. **[Paper 2 Style, Calculator Required, Easy, 4 marks]**
   A biased coin has a probability of landing on tails of 0.38. The coin is tossed exactly 250 times.

   (a) Find the probability that the coin lands on heads on any single toss.

   (b) Calculate the expected number of times the coin will land on tails.

   (c) Calculate the expected number of times the coin will land on heads.

2. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
   Let A and B be two events such that $P(A) = 0.3$ and $P(B) = 0.8$.

   (a) Given that A and B are independent events, find $P(A \cap B)$.

   (b) Hence, find $P(A \cup B)$.

3. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
   In a class of 30 students, 19 play tennis (T), 3 play both tennis and volleyball (V), and 6 do not play either sport [1].

   (a) Draw a fully labelled Venn diagram to represent this information.

   (b) Find the number of students who play volleyball but not tennis.

   (c) Find the probability that a randomly selected student plays volleyball, given that they play tennis.

4. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
A fair four-sided die (with faces numbered 1, 2, 3, 4) and a fair six-sided die (with faces numbered 1, 2, 3, 4, 5, 6) are rolled together. The random variable S represents the sum of the two scores.

   (a) Draw a table of outcomes (sample space diagram) for the sum S.

   (b) Find the probability that the sum S is exactly 7.

   (c) If this pair of dice is rolled 120 times, calculate the expected number of times the sum will be 7.

5. **[Paper 1 Style, Non-Calculator, Medium, 5 marks]**
Let A and B be events such that $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.6$ [2].

   (a) Find $P(A \cap B)$.

   (b) Find $P(A|B)$.

   (c) Determine, with a reason, whether events A and B are independent.

6. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
Saanvi travels to work by either train or bus. The probability she takes the train is 0.6. If she takes the train, the probability she is late is 0.1. If she takes the bus, the probability she is late is 0.25.

   (a) Draw a fully labelled tree diagram representing this scenario.

   (b) Find the probability that Saanvi is late for work on any given day.

   (c) Given that Saanvi is late for work, find the probability that she took the bus.

7. **[Paper 1 Style, Non-Calculator, Medium, 5 marks]**
A bag contains 4 red marbles and 5 blue marbles. Two marbles are drawn from the bag at random, one after the other, **without replacement**.

   (a) Find the probability that both marbles are red.

   (b) Find the probability that the two marbles are of different colours.

8. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
Two events C and D are such that $P(C) = 0.55$ and $P(D) = 0.4$. The probability of neither event occurring is $P(C' \cap D') = 0.27$.

   (a) Find $P(C \cup D)$.

   (b) Find $P(C \cap D)$.

   (c) Find the probability that event C occurs, but event D does not.

9. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
A box contains n green apples and 4 red apples. Two apples are selected at random without replacement. The probability that both apples are green is exactly $\frac{1}{3}$.

   (a) Formulate an algebraic equation in terms of n for the probability of selecting two green apples.

   (b) Use your graphic display calculator to solve the equation and find the exact number of green apples initially in the box.

10. **[Paper 1 Style, Non-Calculator, Hard, 5 marks]**
Let A and B be events such that $P(A) = 0.3$, $P(B) = 0.75$ and $P(A \cup B) = 0.9$ [3].

   (a) Find $P(A \cap B)$.

   (b) Find $P(B|A)$.

   (c) Find $P(A'|B')$.

11. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
A rare disease affects 2% of a population. A diagnostic test is developed. If a person has the disease, the test returns a positive result 95% of the time. If a person does not have the disease, the test incorrectly returns a positive result (a false positive) 10% of the time.

   (a) Find the probability that a randomly selected person tests positive.

   (b) A person tests positive for the disease. Calculate the probability that they actually have the disease.

12. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
A and B are independent events, such that $P(A) = 0.25$ and $P(B) = 0.52$. C is another event, such that B and C are mutually exclusive and $P(A \cap C) = 0.09$. Given that $P(A \cup B \cup C) = 0.95$ [4], find:

   (a) $P(A \cap B)$

   (b) $P(C)$

   (c) $P(A' \cap B')$

   (d) $P(A|C')$

13. **[Paper 1 Style, Non-Calculator, Very Hard, 6 marks]**
Two events A and B exist in a sample space such that $P(A) > 0$ and $P(B) > 0$. A student claims that "if two events are mutually exclusive, then they must also be independent." By using the formal mathematical definitions of mutually exclusive events and independent events, mathematically prove that the student's claim is false.

14. **[Paper 2 Style, Calculator Required, Very Hard, 6 marks]**
A biased coin has a probability p of landing on heads, where $0 < p < 1$. The coin is tossed exactly 3 times. The probability of obtaining exactly two heads is equal to the probability of obtaining exactly one head.

    (a) By using a tree diagram or binomial combinations, formulate an equation in terms of p.

    (b) Solve the equation to find the exact value of p.

15. **[Paper 2 Style, Calculator Required, Very Hard, 7 marks]**
A university requires students to pass a sequential two-stage mathematics test. The probability of a student passing Stage 1 is 0.8. If a student passes Stage 1, the probability they pass Stage 2 is 0.7. If a student fails Stage 1, they are given a remedial class, and the probability they pass Stage 2 becomes 0.4.

    (a) Find the probability that a student passes exactly one of the two stages.

    (b) Given that a student passed Stage 2, find the probability that they failed Stage 1.

    (c) The university expects 1500 students to take the test this year. Calculate the expected number of students who will fail both stages.