IB MATHEMATICS AA HL
AHL TOPIC 3 PRACTICE

Scalar Products, Planes, and Vector Kinematics

Instructions to Candidates

- This practice paper contains **20** questions progressing from Easy to Very Hard.
- Each question indicates whether it is styled for **Paper 1 (No Calculator)** or **Paper 2 (Calculator Allowed)**.
- The paper tests syllabus topics AHL 3.13, 3.14, and 3.17: The scalar (dot) product, vector kinematics and position vectors, equation of planes in Cartesian and parametric form, and angles/intersections between lines and planes.
- Answer all questions, showing all your working clearly.
- Total marks available: **95**.

Difficulty Progression

- **SECTION A (Easy):** Basic scalar product calculation, finding the angle between two 3D vectors, unit vectors, testing points on planes, and constant velocity kinematics.
- **SECTION B (Medium):** Intersections of a line and a plane, distance from a point to a plane, algebraic conversions of plane equations, determining unknown parameters for perpendicular vectors, and the angle between a line and a plane.
- **SECTION C (Hard):** Solving the intersection of three planes, finding the line of intersection between two planes, shortest distance between two moving bodies (kinematics), reflections of points in planes, and formal geometric proofs.

SECTION A: EASY (Fundamentals)

Question 1 (2 Marks) — Paper 1 (No Calculator Allowed)

Find the scalar (dot) product $\mathbf{a} \cdot \mathbf{b}$ where $\mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$.

Question 2 (3 Marks) — Paper 2 (Calculator Allowed)

Find the angle, in degrees, between the vectors $\mathbf{u} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} + 4\mathbf{k}$.

Question 3 (3 Marks) — Paper 1 (No Calculator Allowed)

A plane Π passes through the point $(1, -2, 3)$ and has a normal vector $\mathbf{n} = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$. Write down the Cartesian equation of the plane Π.

Question 4 (3 Marks) — Paper 1 (No Calculator Allowed)

Find a unit vector that acts in the same direction as $\mathbf{c} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$.

Question 5 (3 Marks) — Paper 1 (No Calculator Allowed)

A drone starts at position $(2, 5, -1)$ relative to a control tower and moves with a constant velocity of $\mathbf{v} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ metres per second. Find the position vector of the drone after exactly 4 seconds.

Question 6 (2 Marks) — Paper 1 (No Calculator Allowed)

Determine whether the point $P(3, 1, -2)$ lies on the plane defined by the equation $2x - y + 3z = -1$. Justify your answer.

Question 7 (3 Marks) — Paper 2 (Calculator Allowed)

Two non-zero vectors $\mathbf{p}$ and $\mathbf{q}$ have magnitudes $|\mathbf{p}| = 5$ and $|\mathbf{q}| = 8$. The angle between them is $60°$. Calculate the exact value of the scalar product $\mathbf{p} \cdot \mathbf{q}$.

SECTION B: MEDIUM (Application & Algebraic Methods)

Question 8 (5 Marks) — Paper 1 (No Calculator Allowed)

Find the coordinates of the point of intersection between the line $L : \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and the plane $\Pi : x + 2y - z = 5$.

Question 9 (4 Marks) — Paper 2 (Calculator Allowed)

Find the angle between the two planes $\Pi_1 : 2x - y + z = 4$ and $\Pi_2 : x + y + 2z = 7$. Give your answer in exact radians.

Question 10 (5 Marks) — Paper 2 (Calculator Allowed)

Particle A starts at $(0, 2, 1)$ and travels with velocity $\mathbf{v}_A = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ m/s. Particle B starts at $(3, 0, 0)$ and travels with velocity $\mathbf{v}_B = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ m/s. Find the exact distance between the two particles after $t = 3$ seconds.

Question 11 (5 Marks) — Paper 1 (No Calculator Allowed)

A plane is given in parametric vector form as:

$$\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

Find the Cartesian equation of the plane in the form $ax + by + cz = d$.

Question 12 (5 Marks) — Paper 1 (No Calculator Allowed)

The vectors $\mathbf{u} = \begin{pmatrix} k \\ 2 \\ k - 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} k \\ -3 \\ 2 \end{pmatrix}$ are perpendicular. Find all possible values of the real constant k.

Question 13 (6 Marks) — Paper 2 (Calculator Allowed)

Calculate the angle between the line $L : \mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ and the plane $\Pi : 3x - 4y = 10$.

Give your answer in degrees, correct to one decimal place.

Question 14 (5 Marks) — Paper 2 (Calculator Allowed)

The shortest distance D from a point $P(x_0, y_0, z_0)$ to a plane $ax + by + cz = d$ is given by $D = \frac{|ax_0+by_0+cz_0-d|}{\sqrt{a^2+b^2+c^2}}$. Calculate the exact shortest distance from the point $P(4, -1, 2)$ to the plane $2x + y - 2z = 5$.

SECTION C: HARD / VERY HARD (Synthesis & Proof)

Question 15 (5 Marks) — Paper 2 (Calculator Allowed)

Three planes are defined by the following Cartesian equations:

$$\Pi_1 : x + y + z = 6$$

$$\Pi_2 : 2x - y + z = 3$$

$$\Pi_3 : x + 2y - z = 2$$

Show that the three planes intersect at a single unique point, and find the coordinates of this point.

Question 16 (7 Marks) — Paper 1 (No Calculator Allowed)

Two planes intersect to form a straight line. The equations of the planes are $x - y + z = 4$ and $2x + y - z = 5$. Find the vector equation of their line of intersection in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

Question 17 (7 Marks) — Paper 2 (Calculator Allowed)

Two helicopters are flying in linear paths defined by position vectors (in km) with time t in minutes:

$$\mathbf{r}_A = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_B = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Find the time t at which the two helicopters are closest to each other, and state this exact minimum distance.

Question 18 (7 Marks) — Paper 1 (No Calculator Allowed)

Find the exact coordinates of A', the reflection of the point $A(2, 3, -1)$ in the plane $\Pi : x+y+z = 1$.

Question 19 (7 Marks) — Paper 1 (No Calculator Allowed)

A plane Π contains the line $L : \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ and the point $P(3, 1, 4)$. Find the Cartesian equation of the plane Π.

Question 20 (6 Marks) — Paper 1 (No Calculator Allowed)

Using vector scalar products, prove that the diagonals of a rhombus intersect at right angles.
(Hint: Let the sides of the rhombus be represented by vectors $\mathbf{a}$ *and* $\mathbf{b}$ *where* $|\mathbf{a}| = |\mathbf{b}|$*).*