**Unit 3: 3D Geometry & Right-Angled Trigonometry**
**IB Math AA SL**

*Answer all questions. Show all working where appropriate. For Paper 1 questions, you must use analytical algebraic methods and give exact answers. For Paper 2 questions, use your graphic display calculator (GDC) efficiently and round to 3 significant figures.*

1. **[Paper 1 Style, Non-Calculator, Easy, 5 marks]**
   Points $A$ and $B$ have coordinates $(1, -2, 3)$ and $(5, 4, -1)$ respectively in a 3D Cartesian plane. Point $O$ is the origin $(0, 0, 0)$.

   (a) Find the exact distance between $A$ and $B$.

   (b) Find the coordinates of $M$, the midpoint of $[AB]$.

   (c) Find the exact distance of $OM$.

2. **[Paper 2 Style, Calculator Required, Easy, 6 marks]**
   Two points, $P(2, 1, 3)$ and $Q(5, 2, 6)$, are located on an $xyz$ coordinate grid.

   (a) Find the length of $PQ$ to three significant figures.

   (b) Find the coordinates of the midpoint of $[PQ]$.

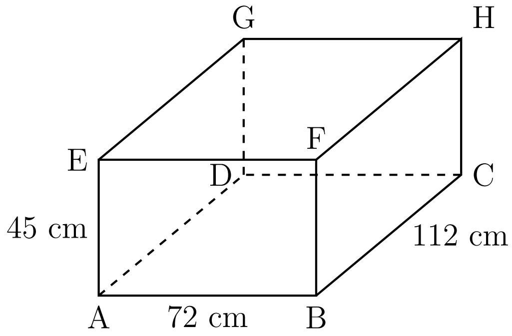
   (c) Calculate the angle between the line segment $[PQ]$ and the horizontal $xy$-plane.

3. **[Paper 1 Style, Non-Calculator, Easy, 5 marks]**
   A right circular cone has a base radius of $r$ and a perpendicular height of $2\sqrt{2}r$.

   (a) Find the exact slant height, $l$, of the cone in terms of $r$.

   (b) Hence, find the exact ratio of the cone's curved surface area to its flat base area.

4. **[Paper 2 Style, Calculator Required, Medium, 6 marks]**
The diagram below shows a cuboid measuring 45 cm × 72 cm × 112 cm.




(a) Calculate the distance from $A$ to $F$.

(b) Calculate the distance of the main interior diagonal, from $A$ to $H$.

(c) Calculate the angle between the diagonal $[AH]$ and the rectangular base $ABCD$.

5. **[Paper 1 Style, Non-Calculator, Medium, 6 marks]**
A solid object is constructed by placing a solid hemisphere of radius $r$ precisely on top of a solid cylinder of radius $r$ and height $h$.

(a) Given that the volume of the hemisphere is equal to the volume of the cylinder, express $h$ in terms of $r$.

(b) Show that the total surface area of the combined solid is exactly $\frac{13}{3}\pi r^2$.

6. **[Paper 2 Style, Calculator Required, Medium, 6 marks]**
A waffle ice cream cone forms a right circular cone that has an interior volume of 120 $\text{cm}^3$ and a radius of 2.8 cm.

(a) Find the vertical height, $h$, of the cone.

(b) Find the slant height, $l$, of the cone.

(c) Calculate the curved surface area of the cone.

7. **[Paper 1 Style, Non-Calculator, Medium, 7 marks]**
A regular tetrahedron is a 3D shape consisting of four identical equilateral triangles. Let the side length of every edge of a particular regular tetrahedron be $2a$.

(a) Show that the exact perpendicular height of the tetrahedron from its base to its top vertex is $\frac{2a\sqrt{6}}{3}$.

(b) Hence, find an exact expression for the volume of the tetrahedron in terms of $a$.

8. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
A model building is created in the shape of a rectangular-based right pyramid $ABCDE$. The base $ABCD$ measures 4.6 cm by 7.2 cm. The slant height of the triangular faces is 8.3 cm.

   (a) Calculate the vertical height of the pyramid.

   (b) Calculate the volume of the model.

   (c) Find the angle that the triangular face $ABE$ (with base 4.6 cm) makes with the rectangular base $ABCD$.

9. **[Paper 1 Style, Non-Calculator, Hard, 5 marks]**
A cube has a side length of $a$. A straight line is drawn representing the main interior diagonal connecting the bottom-front-left corner to the top-back-right corner. Prove analytically that the exact angle, $\theta$, between this main diagonal and any of the square faces of the cube satisfies the equation $\sin\theta = \frac{1}{\sqrt{3}}$.

10. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
A cable-stayed bridge crosses a river from embankment $A$ to embankment $B$. The height of the embankment at $A$, measured from the horizontal river bed, is 9.1 m and this drops to a height of 1.3 m at $B$. The width of the horizontal river bed is 90 m. A vertical central column of height 15 m is situated at the midpoint of the river bed, $P$, and connects to the exterior supporting cables at point $V$. The other ends of the cables are attached at points $A$ and $B$ respectively.

    (a) Calculate the length of the supporting cable $VA$ and the supporting cable $VB$.

    (b) Find the size of the angle $VBA$, between the exterior supporting cable and the bridge span $AB$.

11. **[Paper 1 Style, Non-Calculator, Hard, 7 marks]**
A cuboid $ABCDEFGH$ has a rectangular base $ABCD$ with sides of length $2x$ and $2\sqrt{3}x$. The height of the cuboid is $x$.

    (a) Show that the length of the main diagonal $BH$ can be expressed exactly as $x\sqrt{17}$.

    (b) Find an exact expression for the total surface area of the cuboid, giving your answer in the form $cx^2(1 + d\sqrt{3})$, where $c, d \in \mathbb{Z}$.

12. **[Paper 2 Style, Calculator Required, Very Hard, 7 marks]**
A frustum is made by removing a square-based pyramid of vertical height 4 cm from the top of a larger solid square-based pyramid of vertical height 12 cm and base length 3 cm. The top plane of the frustum is parallel to its base.

    (a) Calculate the exact side length of the top square face of the frustum.

    (b) Calculate the volume of the frustum.

    (c) Calculate the total surface area of the frustum.

13. **[Paper 1 Style, Non-Calculator, Very Hard, 6 marks]**
Sphere $S_1$ has radius $r_1$, volume $V_1$, and surface area $A_1$. Sphere $S_2$ has radius $r_2$, volume $V_2$, and surface area $A_2$. Sphere $S_2$ has exactly eight times the volume of sphere $S_1$.

    (a) Find the exact ratio of the radii, $r_2 : r_1$.

    (b) Find the exact ratio of the surface areas, $A_2 : A_1$.

14. **[Paper 2 Style, Calculator Required, Very Hard, 7 marks]**
A product in the shape of a sphere with radius 3.6 cm is packed into cuboidal crates measuring 1.7 m by 0.9 m with a depth of 22 cm. The spheres are stacked directly on top of and next to each other in a grid formation.

    (a) Find the maximum number of full spheres that can be packed into a single crate.

    (b) Calculate the volume of unused, empty space inside a fully packed crate in $cm^3$.

15. **[Paper 1 Style, Non-Calculator, Very Hard, 7 marks]**
A symmetrical candle in the shape of a right circular cone has a base radius of $r$ and an initial height of $h_1$. As the candle burns, its top is removed parallel to the base, forming a frustum.

    (a) Show that when exactly one-quarter $(\frac{1}{4})$ of the initial volume has been burnt away, the vertical height of the remaining unburnt frustum, $h_2$, is given by $h_2 = \frac{h_1(\sqrt[3]{4}-1)}{\sqrt[3]{4}}$.

    (b) Given that $h_2 = 5$ cm, find the exact height of the conical piece that has burned away.