

Unit 2: Quadratic Functions & Equations IB Math AA SL

Answer all questions. Show all working where appropriate. For Paper 1 questions, you must use analytical algebraic methods. For Paper 2 questions, use your graphic display calculator (GDC) to efficiently locate roots, vertices, and solve polynomials.

1. **[Paper 1 Style, Non-Calculator, Easy, 5 marks]**

The quadratic function $f(x)$ is given in vertex form by $f(x) = -2(x - 3)^2 + 8$.

- Write down the coordinates of the vertex of the graph of f .
- Find the coordinates of the y -intercept.
- Using algebraic methods, find the exact coordinates of the x -intercepts.

2. **[Paper 2 Style, Calculator Required, Easy, 4 marks]**

Consider the quadratic function $g(x) = 3x^2 - 7.2x - 11.5$.

- Use your graphic display calculator to find the coordinates of the x -intercepts, giving your answers to three significant figures.
- Find the coordinates of the local minimum point on the graph of g .

3. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**

The quadratic equation $x^2 + kx + 9 = 0$ has two equal real roots. Using the discriminant ($\Delta = b^2 - 4ac$), find the possible exact values of the constant k .

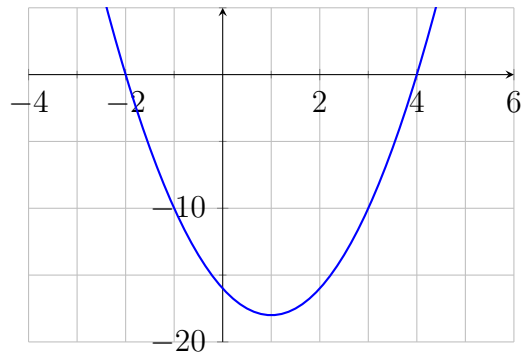
4. **[Paper 2 Style, Calculator Required, Easy, 4 marks]**

The function $h(x) = -0.65x^2 + 5.2x + 3.1$ models the trajectory of an object.

- Sketch the graph of $y = h(x)$ on your graphic display calculator.
- Use the maximum feature of your GDC to find the range of the function $h(x)$.

5. [Paper 1 Style, Non-Calculator, Medium, 6 marks]

The diagram below shows the graph of a quadratic function $y = f(x)$. The graph intersects the x -axis at $(-2, 0)$ and $(4, 0)$, and it intersects the y -axis at $(0, -16)$.



- (a) Find the equation of the function in the factorised form $f(x) = a(x-p)(x-q)$.
- (b) Expand your answer to part (a) to write the function in the standard form $f(x) = ax^2 + bx + c$.

6. [Paper 2 Style, Calculator Required, Medium, 4 marks]

Consider the quadratic curve C with equation $y = 1.8x^2 - 4.5x + 2$ and the straight line L with equation $y = 3.2x - 5$. Using the intersection feature on your graphic display calculator, find the coordinates of the points where C and L intersect, giving your values to three significant figures.

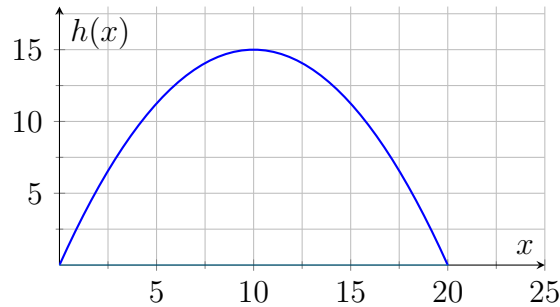
7. [Paper 1 Style, Non-Calculator, Medium, 5 marks]

Let $f(x) = x^2 - 6x + 14$.

- (a) By completing the square, express $f(x)$ in the form $(x - h)^2 + k$.
- (b) Hence, or otherwise, explain algebraically why the graph of $f(x)$ does not intersect the x -axis.

8. [Paper 2 Style, Calculator Required, Medium, 5 marks]

The height of a parabolic bridge arch above the water level can be modelled by the function $h(x) = -0.15x^2 + 3x$, where x is the horizontal distance from the left edge of the bridge.



- Use your GDC to find the maximum height of the bridge above the water.
- Find the total horizontal width of the bridge at water level ($h = 0$).

9. [Paper 1 Style, Non-Calculator, Hard, 5 marks]

Consider the quadratic equation $x^2 + (k + 2)x + 2k = 0$, where $k \in \mathbb{R}$. Show algebraically that this equation will always have real roots, regardless of the value of k .

10. [Paper 2 Style, Calculator Required, Hard, 4 marks]

Solve the quartic equation $2x^4 - 11x^2 + 12 = 0$. (*Hint: You may use a substitution to form a quadratic equation, or use the polynomial solver on your GDC*). Give your answers to three significant figures.

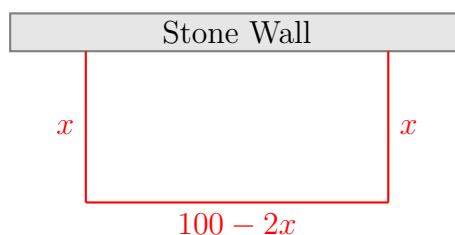
11. [Paper 1 Style, Non-Calculator, Hard, 6 marks]

The straight line $y = mx - 4$ is a tangent to the quadratic curve $y = x^2 + 3x + 5$.

- Set up an equation equating the line and the curve, and write it in the standard quadratic form $ax^2 + bx + c = 0$.
- Using the discriminant, find the two possible exact values of m .

12. [Paper 2 Style, Calculator Required, Hard, 5 marks]

A farmer has 100 m of fencing to create a rectangular enclosure. He uses an existing straight stone wall for one side of the enclosure, meaning he only needs to fence three sides.



- (a) Given that the width of the enclosure is x metres, show that the area, A , is given by $A(x) = 100x - 2x^2$.
- (b) Graph $A(x)$ on your GDC. Find the value of x that maximises the area, and state this maximum area.

13. [Paper 1 Style, Non-Calculator, Very Hard, 7 marks]

The graph of a quadratic function $f(x) = ax^2 + bx + c$ has its vertex at $(2, -3)$ and passes through the point $(4, 5)$. By first writing the equation in vertex form, determine the exact values of the constants a , b , and c .

14. [Paper 2 Style, Calculator Required, Very Hard, 7 marks]

The height of a thrown ball, h in metres, t seconds after it is released is modelled by the equation $h(t) = -4.9t^2 + 12t + 5$.

- (a) Use your graphic display calculator to find the exact times when the ball is at a height of exactly 10 m.
- (b) Hence, calculate the total length of time that the ball remains above 10 m in the air. Give your answer to three significant figures.

15. [Paper 1 Style, Non-Calculator, Very Hard, 7 marks]

Let $f(x) = x^2 - (p + 2)x + (p + 5)$, where $p \in \mathbb{R}$.

- (a) Find an expression for the discriminant of $f(x)$ in terms of p , fully simplified.
- (b) Find the exact values of p for which the graph of $f(x)$ touches the x -axis at exactly one point.