Topic 1: Systems of Linear Equations
IB Math AI SL

Answer all questions. Show all working where appropriate. Solutions found from a Graphic Display Calculator (GDC) should be supported by suitable working. Total: 68 marks.

1. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
   A theme park sells adult tickets for $\$x$ and child tickets for $\$y$. A family of 3 adults and 5 children pays \$165. Another family of 2 adults and 3 children pays \$105 [1].

   (a) Write down a system of two linear equations representing this information.

   (b) Using the simultaneous equation solver on your Graphic Display Calculator (GDC), find the price of one adult ticket and the price of one child ticket.

2. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
   Consider the following system of three linear equations [2]:

$$\begin{aligned} a + b + c &= 8 \\ 8a + 4b + 2c &= 4 \\ 27a + 9b + 3c &= 0 \end{aligned}$$

   (a) Write down the augmented matrix or the system of equations as it would be entered into your GDC.

   (b) Using your GDC, find the exact values of a, b, and c.

3. **[Paper 1 Style, Short Answer, Easy, 6 marks]**
A local bakery makes three types of cakes: Chocolate, Vanilla, and Lemon [3].

- On Monday, they sold 5 Chocolate, 3 Vanilla, and 2 Lemon cakes for a total revenue of \$158.
- On Tuesday, they sold 4 Chocolate, 6 Vanilla, and 1 Lemon cake for a total revenue of \$178.
- On Wednesday, they sold 2 Chocolate, 5 Vanilla, and 4 Lemon cakes for a total revenue of \$179.

(a) Let c, v, and l represent the price of a Chocolate, Vanilla, and Lemon cake, respectively. Write down a system of three linear equations to represent the sales data.

(b) Using your GDC, find the exact price of each type of cake.

4. **[Paper 1 Style, Short Answer, Medium, 6 marks]**
The ticket prices for a concert are \$15 for an adult, \$10 for a child, and \$12 for a student. A total of 600 tickets were sold. The total amount of money from ticket sales was \$7816. There were twice as many adult tickets sold as child tickets [4]. Let the number of adult tickets sold be x, the number of child tickets sold be y, and the number of student tickets sold be z.

(a) Write down three linear equations that express the information given above.

(b) Using your GDC, find the exact number of adult, child, and student tickets sold.

5. **[Paper 1 Style, Short Answer, Medium, 6 marks]**
The graph of a quadratic function $y = ax^2 + bx + c$ passes through the points $(0, 10)$, $(4, 12)$, and $(8, 10)$ [5].

(a) Use the point $(0, 10)$ to write down the value of c.

(b) Write down two linear equations in terms of a and b using the other two points.

(c) Using the simultaneous equation solver on your GDC, find the values of a and b.

6. **[Paper 1 Style, Short Answer, Medium, 6 marks]**
A dietician is planning a meal consisting of three foods: Food A, Food B, and Food C.

- One serving of Food A contains 2 g of protein, 3 g of carbohydrates, and 1 g of fat.
- One serving of Food B contains 1 g of protein, 2 g of carbohydrates, and 2 g of fat.
- One serving of Food C contains 3 g of protein, 1 g of carbohydrates, and 2 g of fat.

The meal must provide exactly 16 g of protein, 19 g of carbohydrates, and 15 g of fat.

(a) Write down a system of three equations to find the number of servings of each food type required.

(b) Using your GDC, find the number of servings needed for Food A, Food B, and Food C.

7. **[Paper 1 Style, Short Answer, Hard, 6 marks]**
A polynomial function is given by $f(x) = ax^3 + bx^2 + cx$. It is known that the graph of the function passes through the points $(1, 2)$, $(2, 6)$, and $(-1, -6)$.

(a) Set up a system of three linear equations in terms of a, b, and c.

(b) Using your GDC, solve the system to find the values of a, b, and c.

(c) Hence, find the value of $f(3)$.

8. **[Paper 2 Style, Longer Question, Hard, 8 marks]**
A financial advisor invests a total of \$50 000 across three different accounts: Account A, Account B, and Account C.

- Account A earns 3% simple interest per year.
- Account B earns 4% simple interest per year.
- Account C earns 5% simple interest per year.

After one year, the total interest earned from all three accounts is \$2050. Additionally, the amount invested in Account A is exactly twice the amount invested in Account B.

(a) Let A, B, and C represent the amounts invested in each account. Write down a system of three linear equations.

(b) Using your GDC, find the exact amount invested in each of the three accounts.

(c) If the entire \$50 000 had been distributed equally among the three accounts, calculate what the total interest earned would have been.

9. **[Paper 2 Style, Longer Question, Very Hard, 9 marks]**
M-Line is a company that prints custom designs on T-shirts. They charge a fixed design fee plus an additional fee for each T-shirt printed. The total charge, M, in euros, is modelled by $M(x) = 5x + 40$ where x is the number of T-shirts ordered [6]. A rival company, N-Line, models their total charge by $N(x) = 4.5x + 60$ [6]. A third company, P-Line, models their total charge by the linear function $P(x) = px + q$.

   (a) Find the exact number of T-shirts, x, for which M-Line and N-Line charge exactly the same amount, and find this cost.

   (b) It is known that P-Line also charges this exact same amount for that same number of T-shirts. Furthermore, P-Line charges \$150 for 10 T-shirts. Write down a system of two linear equations in terms of p and q.

   (c) Using your GDC, solve the system to find the values of p and q.

10. **[Paper 2 Style, Longer Question, Very Hard, 12 marks]**
An engineer is tracking a drone in 3D space. The drone hovers at a specific coordinate (x, y, z). It receives signals from three different planar laser sensors, producing the following system of linear equations based on its position:

$$\begin{aligned} x + 2y - z &= 4 \\ 2x - y + 3z &= 13 \\ -x + 3y + 2z &= 15 \end{aligned}$$

   (a) Write down the augmented matrix for this system.

   (b) Using the simultaneous equation solver on your GDC, find the exact coordinates (x, y, z) of the drone.

   (c) The origin is at $(0, 0, 0)$. Calculate the exact straight-line distance from the origin to the drone.

   (d) A second drone is located exactly at the point $(-1, 4, 2)$. Show that this second drone satisfies exactly two of the three planar equations.