Topic 1: Financial Mathematics
IB Math AI SL

Answer all questions. Show all working where appropriate. Solutions found from a Graphic Display Calculator (GDC) should be supported by suitable working (e.g., writing down the financial solver parameters entered). Total: 104 marks.

1. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
   David invests \$6000 in a bank account that pays 3.5% p.a. **simple interest** for 4 years.

   (a) Calculate the total interest earned over the 4 years.

   (b) Calculate the total value of the investment at the end of the 4 years.

2. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
   Sarah invests \$8000 in a savings account that pays a nominal annual interest rate of 4.2%, **compounded quarterly** for 5 years.

   (a) Write down the values you would enter into the financial application on your GDC (i.e., N, $I\%$, PV, P/Y, C/Y).

   (b) Calculate the future value of the investment.

   (c) Calculate the total interest earned.

3. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
   A company purchases a new delivery van for \$32 000. The value of the van depreciates at a rate of 14% per annum.

   (a) Find the value of the van after exactly 3 years.

   (b) Using your GDC, find the exact time (in years) it takes for the van's value to halve.

4. [**Paper 1 Style, Short Answer, Easy, 5 marks**]
Liam invests \$5000 into an account that pays a nominal annual interest rate of 5.4%, **compounded monthly**. He wants his investment to grow to \$8000.

   (a) Using your GDC, find the exact time (in years) required to reach \$8000.

   (b) Determine the number of **full months** Liam must wait for his account to reach or exceed \$8000.

5. [**Paper 1 Style, Short Answer, Medium, 6 marks**]
Isabella invests \$15 000 in an account paying 6.5% nominal annual interest, **compounded annually** for 8 years. The average rate of inflation over this period is expected to be 2.4% per year.

   (a) Calculate the nominal value of the investment after 8 years.

   (b) Calculate the approximate real interest rate.

   (c) Calculate the real value of the investment after 8 years, taking inflation into account.

6. [**Paper 1 Style, Short Answer, Medium, 5 marks**]
Noah invests \$10 000 in a high-yield account. After exactly 7 years, the investment has grown to \$14 500. The interest is **compounded half-yearly**.

   (a) Write down the equation, or the GDC financial app inputs, that can be used to solve for the interest rate.

   (b) Find the nominal annual interest rate.

7. [**Paper 2 Style, Longer Question, Medium, 7 marks**]
Elena makes a deposit of \$400 at the **end of each month** into a savings annuity paying a nominal annual rate of 4.8%, **compounded monthly**. She does this for 10 years.

   (a) Using your GDC, calculate the total future value of the account after 10 years.

   (b) Calculate the total amount of money Elena deposited out of her own pocket.

   (c) Calculate the total interest Elena earned on her annuity over the 10 years.

8. **[Paper 1 Style, Short Answer, Medium, 6 marks]**
A financial institution offers two different investment plans for a \$25 000 principal over 4 years:

- **Option A:** 5.5% p.a. compounded annually.
- **Option B:** 5.3% p.a. compounded monthly.

(a) Calculate the final value of the investment under Option A.

(b) Calculate the final value of the investment under Option B.

(c) State which option is better and calculate the difference in their final values.

9. **[Paper 2 Style, Longer Question, Hard, 8 marks]**
A family takes out a mortgage of \$200 000 to buy a house. The loan is amortized over 20 years at a nominal annual interest rate of 4.5%, **compounded monthly**. Repayments are made at the end of each month.

(a) Using the financial app on your GDC, calculate the fixed monthly repayment amount.

(b) Calculate the total amount the family will pay to the bank over the 20 years.

(c) Calculate the total interest paid to the bank over the life of the loan.

10. **[Paper 2 Style, Longer Question, Hard, 9 marks]**
Consider the same mortgage from Question 9: A \$200 000 loan over 20 years at 4.5% p.a. **compounded monthly**, with fixed monthly repayments.

(a) Using your GDC, calculate the outstanding balance (the amount still owed) at the end of exactly 5 years.

(b) Calculate the amount of the principal that has been paid off during the first 5 years.

(c) Calculate the total interest paid during the first 5 years.

11. **[Paper 2 Style, Longer Question, Hard, 8 marks]**
A retiree has \$600 000 in a secure pension fund paying a nominal annual interest rate of 3.8%, **compounded monthly**.

(a) If the retiree wants the money to last for exactly 25 years (until the balance is exactly zero), find the fixed amount they can withdraw at the end of each month.

(b) If instead they decide to withdraw exactly \$4000 at the end of each month, find the number of **full months** the fund will last.

(c) In the scenario described in part (b), calculate the total amount of money withdrawn before the final partial month.

12. [**Paper 1 Style, Short Answer, Hard, 6 marks**]
James invests \$10 000 in an account paying 4% p.a. **compounded quarterly** for 3 years. At the end of the 3 years, he transfers the entire accumulated amount into a new account paying 5% p.a. **compounded monthly** for another 2 years.

   (a) Find the value of the investment at the end of the first 3 years.

   (b) Find the final value of the investment at the end of the 5 years.

   (c) Calculate the overall percentage increase of the original \$10 000 investment over the 5-year period.

13. [**Paper 2 Style, Longer Question, Very Hard, 8 marks**]
John plans to deposit \$500 at the end of each year into an annuity paying a nominal annual interest rate of 6.0%, **compounded annually** for 10 years. He expects the average inflation rate over this period to be 2.5% per year.

   (a) Calculate the nominal future value of the annuity after 10 years.

   (b) Calculate the approximate real interest rate.

   (c) Calculate the real value of the annuity after 10 years by using the real interest rate in your GDC calculations.

14. [**Paper 2 Style, Longer Question, Very Hard, 9 marks**]
An industrial printer costs \$80 000 new. Its value depreciates at a rate of 20% per year. The maintenance cost to keep the printer running is \$1000 in the first year, and this cost increases by 15% each subsequent year.

   (a) Calculate the value of the printer at the end of year 5.

   (b) Calculate the maintenance cost during year 5.

   (c) Calculate the total accumulated maintenance cost over the first 5 years.

   (d) Using your GDC, find the year, n, in which the annual maintenance cost first strictly exceeds the value of the printer at the end of that year.

15. [**Paper 2 Style, Longer Question, Very Hard, 13 marks**]
A family takes out a loan of \$300 000 for 25 years at a nominal annual interest rate of 4.8%, **compounded monthly**.

(a) Find their required fixed monthly payment.

After exactly 10 years of making this regular payment, the bank drops the interest rate to 3.6% p.a., **compounded monthly**.

(b) Calculate the outstanding balance of the loan at the end of the 10 years (just before the rate drops).

(c) The family decides to keep paying the **exact same monthly payment** found in part (a), despite the interest rate drop. Using your GDC, calculate the exact number of months, n, it will take to pay off the remaining balance.

(d) Determine how many months early they will pay off the 25-year loan by keeping the higher payments.

(e) Calculate the total amount of money they paid to the bank over the entire life of the loan under these new conditions.