Unit 4: The Normal Distribution
IB Math AA SL

Answer all 15 questions. Show all working. For Paper 1 questions, use analytical methods and the properties of the normal curve. For Paper 2 questions, use your graphic display calculator (GDC) efficiently.

1. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
   The distribution of heights of adult women in the UK follows a normal distribution with a mean of 162 cm and a standard deviation of 6.3 cm. Using the geometric properties of the normal distribution curve (the 68-95-99.7 rule), calculate the approximate range of heights within which:

   (a) the central 68% of adult women will fall.

   (b) the central 95% of adult women will fall.

2. **[Paper 2 Style, Calculator Required, Easy, 4 marks]**
   The weight, W in grams, of a chocolate bar produced by a certain manufacturer is modelled as $W \sim N(200, 1.75^2)$.

   (a) Find $P(W < 195)$.

   (b) The manufacturer produces a batch of 500 chocolate bars. Calculate the expected number of chocolate bars from this batch that will weigh more than 203 g.

3. **[Paper 2 Style, Calculator Required, Easy, 4 marks]**
   The weight, P in kg, of pumpkins purchased from a farm is normally distributed with a mean of 11.3 kg and a standard deviation of 2.1 kg. The heaviest 7% of pumpkins are classified as "premium large" and are sold at a higher price. Find the minimum weight a pumpkin must be to be classified as premium large.

4. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
A scientist is studying the movement of snails and has observed that the distribution of their speeds, S, follows a normal distribution with a mean of 48 m/h and a standard deviation of 1.5 m/h.

   (a) Find the probability that a randomly selected snail has a speed of less than 46.5 m/h.

   (b) From a random sample of 80 snails, calculate the expected number of snails that would have a speed of less than 46.5 m/h. Give your answer to the nearest integer.

5. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
For the standard normal distribution $Z \sim N(0, 1^2)$, find:

   (a) $P(Z < 1.5)$.

   (b) $P(-2.1 < Z < -0.3)$.

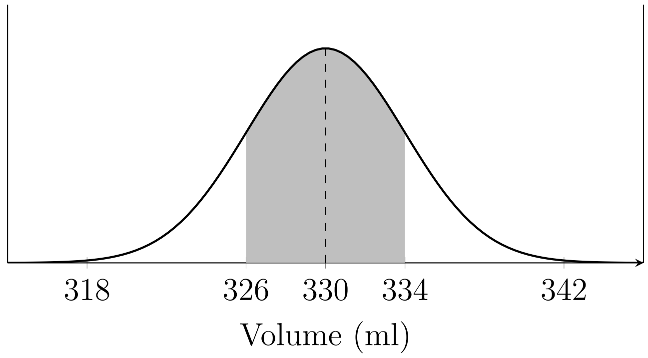
   (c) A random variable is given as $X \sim N(2, 0.1^2)$. By using the standardization formula, re-express the probability $P(X < 2.15)$ in the form $P(Z < a)$, where a is a constant to be found.

6. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
The ages, A in years, that professional football players make their debut in a major league are normally distributed with a mean of 22.5 years and a standard deviation of σ years. Given that 10% of players make their debut before turning 20 years old:

   (a) Find the value of σ.

   (b) Find the probability that a randomly selected player made their debut before their 18^{th} birthday.

7. **[Paper 1 Style, Non-Calculator, Medium, 4 marks]**
The graph below shows the normal distribution of the volume, in ml, of drink in a can provided by a manufacturer. The central 68% of the distribution has been shaded.



Using your knowledge of the properties of the normal curve, strictly deduce:

(a) The mean volume of a can.

(b) The standard deviation of the volume.

8. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
The weights, W in kg, of coconuts grown on a plantation are modelled as a normal distribution with a mean of 1.25 kg and a standard deviation of 0.38 kg. The plantation only considers coconuts to be exportable if their weight falls strictly into the 20% to 80% interpercentile range. Find the range of possible weights, to the nearest 0.01 kg, for an exportable coconut.

9. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
A machine is used to fill cans of a particular brand of soft drink. The volume, V in ml, of soft drink in the cans is normally distributed with a mean of 330 ml and a standard deviation of σ ml.

(a) Given that exactly 15% of the cans contain more than 333.4 ml of soft drink, find the value of σ.

(b) Find $P(320 \leq V \leq 340)$.

(c) Six cans of the soft drink are chosen at random. Find the probability that all six cans contain less than 329 ml of soft drink.

10. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
The stem heights, H in cm, of a particular variety of tulip follow a normal distribution where $H \sim N(60.1, 57.76)$.

    (a) Write down the standard deviation of the tulip heights.

    (b) A tulip is selected at random. Given that it is known that the height of its stem is strictly more than 62 cm, find the probability that the stem height is taller than 64 cm.

11. **[Paper 1 Style, Non-Calculator, Hard, 5 marks]**
A continuous random variable follows a normal distribution $X \sim N(\mu, \sigma^2)$. The probability that X is less than a certain value a is given by $P(X < a) = p$, where $a < \mu$. Using the geometric symmetry of the normal distribution curve, express $P(X > 2\mu - a)$ in terms of p. Justify your answer clearly.

12. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
A reaction time test measures how quickly a person can press the space bar on a computer after a signal. The results, T in milliseconds, are distributed normally with a mean of 273 ms and a variance of 121 ms^2. A result that lies outside of 2.5 standard deviations from the mean is considered to be "extreme".

    (a) Find the exact probability that a randomly selected person has an extreme reaction time.

    (b) A group of 145 students decide to measure their reaction times. Estimate the number of students that will receive a result that would be considered extreme. Give your answer to the nearest integer.

13. **[Paper 2 Style, Calculator Required, Very Hard, 7 marks]**
The random variable X represents the test scores of students in a national exam and is normally distributed such that $X \sim N(\mu, \sigma^2)$. It is known that $P(X > 36.88) = 0.025$ and $P(X < 27.16) = 0.1$.

    (a) Use the inverse normal function on your GDC to find the z-values corresponding to the probabilities above.

    (b) Hence, formulate two simultaneous linear equations in terms of μ and σ.

    (c) By solving these simultaneous equations, determine the exact values of μ and σ.

14. **[Paper 1 Style, Non-Calculator, Very Hard, 5 marks]**
A continuous random variable $X \sim N(\mu, \sigma^2)$. The standardized variable is given by the transformation $Z = \frac{X-\mu}{\sigma}$. Given that the expected value $E(X) = \mu$ and the variance $\text{Var}(X) = \sigma^2$: Use the linear transformation rules for expectation $E(aX + b)$ and variance $\text{Var}(aX + b)$ to formally prove that the standard normal variable Z has a mean of exactly 0 and a variance of exactly 1.

15. [**Paper 2 Style, Calculator Required, Very Hard, 8 marks**]
The running time of feature films at a cinema is normally distributed with a mean time of 102 minutes and a standard deviation of 13 minutes.

(a) Find the probability that a randomly selected feature film has a running time of more than 99 minutes.

(b) Over the course of a month, Jonah watches exactly 18 different feature films. Find the expected number of occasions on which the film he watches will last less than 95 minutes.

(c) Find the probability that on at least 6 out of the 18 occasions, the film will last for longer than 99 minutes.