Unit 5: Integration Techniques
IB Math AA SL

Answer all 15 questions. Show all working. For Paper 1 questions, use analytical algebraic methods. For Paper 2 questions, use your graphic display calculator (GDC) efficiently.

1. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
   Find the indefinite integral for each of the following expressions:
   (a) $\int (6x^2 - 4x + 3)\,dx$ [4]
   (b) $\int \left(\frac{2}{x} + e^x\right)\,dx$

2. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
   The gradient function of a curve is given by $\frac{dy}{dx} = 4x^3 - 2x$. The curve passes through the point $(2, 10)$.
   (a) Find the general expression for y by integrating the gradient function.
   (b) Use the given boundary condition to find the constant of integration, C, and write down the exact equation of the curve.

3. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
   Find the indefinite integral:

$$\int (3\sin x - 2\cos x)\,dx$$

4. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
   The derivative of a function f is given by $f'(x) = e^{5x} - 2\cos(2x)$.
   (a) Find the general anti-derivative, $f(x)$.
   (b) Given that $f(0) = 3$, find the exact value of the constant of integration.

5. **[Paper 1 Style, Non-Calculator, Medium, 5 marks]**
Given that $f(x) = 2x^3 + 4x$, find $f'(x)$. [5] Hence, or otherwise, find the indefinite integral:

$$\int \frac{3x^2 + 2}{2x^3 + 4x} \, dx$$

6. **[Paper 2 Style, Calculator Required, Medium, 6 marks]**
The marginal cost of producing x units of a product, in dollars per unit, is given by the derivative $C'(x) = 0.6x^2 - 4x + 20$. [6]

   (a) Find an expression for the total cost function, $C(x)$, given that the fixed costs (the cost when $x = 0$) are \$150.

   (b) Use your total cost function to calculate the exact cost of producing 10 units.

7. **[Paper 1 Style, Non-Calculator, Medium, 4 marks]**
By using integration by inspection (the reverse chain rule) on linear composites, find the indefinite integral: [7]

$$\int (3x - 1)^3 \, dx$$

8. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
A curve has a gradient function given by:

$$\frac{dy}{dx} = \sin\left(2x - \frac{\pi}{3}\right)$$

Find an expression for y given the boundary condition that $y = 4$ when $x = \frac{\pi}{6}$. [8]

9. **[Paper 1 Style, Non-Calculator, Hard, 5 marks]**
Find an expression for y given that: [9]

$$\frac{dy}{dx} = xe^{x^2-2}$$

and the curve passes through the point $(-\sqrt{2}, 3)$.

10. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
A particle moves in a straight line such that its velocity, v m/s, at time t seconds is given by $v(t) = 3t^2 - 8t$ for $t \geq 0$. [10]

    (a) Find an expression for the displacement function $s(t)$, given that the initial displacement of the particle is $s(0) = 5$ m.

    (b) Calculate the exact displacement of the particle after 4 seconds.

11. **[Paper 1 Style, Non-Calculator, Hard, 5 marks]**
Let $f'(x) = x^2 \cos(x^3 + 1)$. [11] Find the specific function $f(x)$ given the boundary condition $f(-1) = 1$.

12. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
Water is leaking from a tank at a rate given by $R(t) = 50e^{-0.2t}$ litres per minute, where t is the time in minutes since the leak began. [12] Let $V(t)$ represent the total volume of water that has leaked out of the tank after t minutes.

    (a) By interpreting $R(t)$ as the derivative $V'(t)$, find an expression for $V(t)$, given that $V(0) = 0$.

    (b) Hence, calculate the total volume of water leaked after exactly 10 minutes, giving your answer to 3 significant figures.

13. **[Paper 1 Style, Non-Calculator, Very Hard, 6 marks]**
Consider the function $f(x) = \ln(2x^2 + 1)$. [11]

    (a) Find $f'(x)$.

    (b) Hence, by recognising the reverse chain rule, evaluate the indefinite integral:

$$\int \frac{x}{2x^2 + 1}\, dx$$

14. **[Paper 2 Style, Calculator Required, Very Hard, 7 marks]**
A remote-controlled car travels in a straight line. Its acceleration, a m/s^2, is given by $a(t) = e^{0.5t}$ for $t \geq 0$. The car has an initial velocity of 2 m/s and an initial displacement of 0 m from its starting position.

    (a) Find an expression for the velocity, $v(t)$, using the given initial condition.

    (b) Find an expression for the displacement, $s(t)$, using the given initial condition.

    (c) Calculate the displacement of the car when $t = 4$ seconds.

15. **[Paper 1 Style, Non-Calculator, Very Hard, 7 marks]**
The second derivative of a curve is given by $f''(x) = 6x - \frac{4}{x^2}$ for $x > 0$. The curve has a gradient of 2 at the point where $x = 1$. The curve passes through the point $(1, 5)$.

    (a) Integrate $f''(x)$ to find an expression for the first derivative $f'(x)$.

    (b) Integrate $f'(x)$ to find the specific equation of the curve $y = f(x)$.