IB MATHEMATICS AA HL
AHL TOPIC 3 PRACTICE

Reciprocal & Inverse Trig Functions, Advanced Identities

Instructions to Candidates

- This practice paper contains **20** questions progressing from Easy to Very Hard.
- Each question indicates whether it is styled for **Paper 1 (No Calculator)** or **Paper 2 (Calculator Allowed)**.
- The paper tests syllabus topics AHL 3.9 and 3.10: Secant, cosecant, and cotangent definitions, Pythagorean identities, arcsin/arccos/arctan domains and graphs, compound angle identities, and double angle identities.
- Answer all questions, showing all your working clearly.
- Total marks available: **95**.

Difficulty Progression

- **SECTION A (Easy):** Exact evaluations of reciprocal functions, domain and range of inverse trig functions, simplifying using Pythagorean identities, and basic compound/-double angle formulas.
- **SECTION B (Medium):** Proving trigonometric identities algebraically, evaluating expressions like $\cos(\arctan x)$, graphing inverse functions with transformations, and solving equations with reciprocal functions.
- **SECTION C (Hard):** Formal derivation of compound identities, solving simultaneous inverse trig equations, complex fractional identities ($\frac{\sin 3\theta}{\sin \theta}$), and solving quadratic reciprocal trigonometric equations.

SECTION A: EASY (Fundamentals)

Question 1 (3 Marks) — Paper 1 (No Calculator Allowed)

Find the exact value of $\sec\left(\frac{\pi}{3}\right) + \csc\left(\frac{\pi}{6}\right)$.

Question 2 (2 Marks) — Paper 1 (No Calculator Allowed)

State the domain and the range of the inverse trigonometric function $f(x) = \arccos(x)$.

Question 3 (3 Marks) — Paper 1 (No Calculator Allowed)

Simplify the expression $\frac{\tan^2\theta + 1}{\sec^2\theta}$.

Question 4 (4 Marks) — Paper 1 (No Calculator Allowed)

Find all exact solutions to the equation $\sec^2 x = 4$ in the interval $0 \leq x \leq 2\pi$.

Question 5 (4 Marks) — Paper 1 (No Calculator Allowed)

By expanding $\cos(45° + 30°)$ using the appropriate compound angle identity, find the exact value of $\cos 75°$.

Question 6 (2 Marks) — Paper 1 (No Calculator Allowed)

Evaluate exactly: $\arcsin\left(\sin\left(\frac{2\pi}{3}\right)\right)$.

Question 7 (3 Marks) — Paper 1 (No Calculator Allowed)

Given that $\tan A = 2$, use a double angle identity to find the exact value of $\tan 2A$.

SECTION B: MEDIUM (Application & Algebraic Methods)

Question 8 (4 Marks) — Paper 1 (No Calculator Allowed)

Prove the trigonometric identity:

$$\frac{\cos x}{1-\sin x} \equiv \sec x + \tan x$$

Question 9 (5 Marks) — Paper 1 (No Calculator Allowed)

Given that $\sin A = \frac{3}{5}$ where A is an acute angle, and $\cos B = -\frac{5}{13}$ where B is an obtuse angle, find the exact value of $\sin(A+B)$.

Question 10 (5 Marks) — Paper 1 (No Calculator Allowed)

Solve the equation $2\sec^2 x - \tan^2 x = 5$ for exact values of x in the interval $0 \leq x \leq 2\pi$.

Question 11 (4 Marks) — Paper 1 (No Calculator Allowed)

Without using a calculator, find the exact value of $\cos\left(\arctan\frac{4}{3}\right)$.

Question 12 (3 Marks) — Paper 1 (No Calculator Allowed)

Starting from the compound angle identity for $\cos(A+B)$, derive the double angle identity $\cos 2A = 1 - 2\sin^2 A$.

Question 13 (6 Marks) — Paper 1 (No Calculator Allowed)

Consider the function $y = \arcsin(x-1)$.

(a) State the domain and range of the function. **[3 marks]**

(b) Sketch the graph of the function on a Cartesian plane, clearly labelling the endpoints. **[3 marks]**

Question 14 (6 Marks) — Paper 1 (No Calculator Allowed)

Solve the equation $\sin\left(x+\frac{\pi}{6}\right) = 2\cos x$ for $0 \leq x \leq 2\pi$. Give your answers exactly.

SECTION C: HARD / VERY HARD (Synthesis & Proof)

Question 15 (5 Marks) — Paper 1 (No Calculator Allowed)

Using the compound angle identities for sine and cosine, prove that:

$$\tan(A+B) \equiv \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Question 16 (6 Marks) — Paper 1 (No Calculator Allowed)

Find the exact value of $\tan 15°$, giving your answer in the form $a - \sqrt{b}$ where a and b are integers.

Question 17 (7 Marks) — Paper 1 (No Calculator Allowed)

Solve the following equation algebraically for x:

$$\arcsin x + \arccos(2x) = \frac{\pi}{2}$$

Question 18 (6 Marks) — Paper 1 (No Calculator Allowed)

Prove the following trigonometric identity:

$$\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} \equiv 2$$

Question 19 (4 Marks) — Paper 1 (No Calculator Allowed)

Let $f(x) = \arccos\left(\frac{x}{2}\right)$ and $g(x) = \sec x$. Find the exact value of the composite function $g(f(1))$.

Question 20 (7 Marks) — Paper 2 (Calculator Allowed)

Solve the equation $3\csc^2 x + \cot x = 5$ for $0 \leq x \leq \pi$. Give your answers correct to three significant figures.