Unit 2: Exponentials and Logarithms (Analytical & Graphical)
IB Math AA SL

Answer all questions. Show all working where appropriate. For Paper 1 style questions, you must use analytical algebraic methods. For Paper 2 style questions, use your graphic display calculator (GDC) to efficiently locate roots, asymptotes, and intersections.

1. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
   Solve the following equations exactly:

   (a) $e^{3x-1} = 20$

   (b) $\ln(2x) = 5$

2. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
   Let the function f be defined by $f(x) = e^x - 3x$.

   (a) Use your graphic display calculator to find the coordinates of the local minimum point.

   (b) Write down the exact coordinates of the y-intercept.

   (c) Explain why the equation $e^x - 3x = 0$ has no real solutions.

3. **[Paper 1 Style, Non-Calculator, Easy, 5 marks]**
   Given that $\log_a 3 = x$ and $\log_a 4 = y$. Write down expressions for the following in terms of x and y:

   (a) $\log_a 12$

   (b) $\log_a 0.75$

   (c) $\log_a 36$

4. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
Consider the functions $f(x) = \ln x$ and $g(x) = 5 - x$.

   (a) Sketch both graphs on your graphic display calculator for $x > 0$.

   (b) Find the exact coordinates of the x-intercept of $f(x)$ and $g(x)$.

   (c) Use the intersection feature on your GDC to find the solution to the equation $\ln x = 5 - x$, giving your answer to three significant figures.

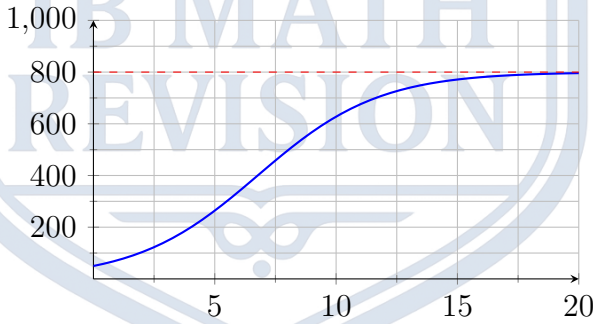
5. **[Paper 1 Style, Non-Calculator, Medium, 5 marks]**
Solve the exponential equation analytically by expressing both sides with a common base:

$$9^{x-2} = \left(\frac{1}{27}\right)^{x+1}$$

6. **[Paper 2 Style, Calculator Required, Medium, 6 marks]**
A population of rabbits on an island is modelled by the logistic growth function:

$$P(t) = \frac{800}{1 + 15e^{-0.4t}}$$

where t is the time in years since the population was first recorded.



   (a) Find the initial population of rabbits.

   (b) Write down the equation of the horizontal asymptote and interpret its meaning in this context.

   (c) Use your GDC to find the time it takes for the population to reach 400 rabbits.

7. **[Paper 1 Style, Non-Calculator, Medium, 6 marks]**
Solve the following logarithmic equation analytically:

$$\log_2 x + \log_2(x - 6) = 4$$

8. **[Paper 1 Style, Non-Calculator, Medium, 5 marks]**
Let the functions f and g be defined as $f(x) = e^x$ and $g(x) = 2\ln x$ for $x > 0$.

   (a) Find an expression for $(f \circ g)(x)$, fully simplified.

   (b) Find an expression for $(g \circ f)(x)$, fully simplified.

9. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
The radioactive mass, M in grams, of a substance after t days is given by $M(t) = 40e^{-kt}$, where k is a positive constant.

   (a) Given that the mass halves after exactly 12 days, use your GDC's equation solver to find the value of k to four significant figures.

   (b) Calculate the mass remaining after 30 days.

10. **[Paper 1 Style, Non-Calculator, Hard, 6 marks]**
Use the change of base formula to solve the following equation analytically:

$$\log_4 x + \log_{16} x = 3$$

11. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
Let $f(x) = \ln(x^2 - 4)$.

   (a) State the maximal valid domain for $f(x)$.

   (b) Write down the equations of the two vertical asymptotes.

   (c) Use your GDC to find the exact coordinates of the x-intercepts.

12. **[Paper 1 Style, Non-Calculator, Hard, 6 marks]**
Solve the following exponential equation analytically:

$$e^{2x} - 8e^x + 15 = 0$$

(Hint: use a substitution to form a quadratic equation). Give your answers in exact logarithmic form.

13. **[Paper 2 Style, Calculator Required, Hard, 7 marks]**
Consider the functions $f(x) = 2xe^{-x}$ and $g(x) = 0.5x^2$.

   (a) Sketch both graphs on your GDC for $-1 \leq x \leq 3$.

   (b) Find the coordinates of the local maximum point on the graph of $f(x)$.

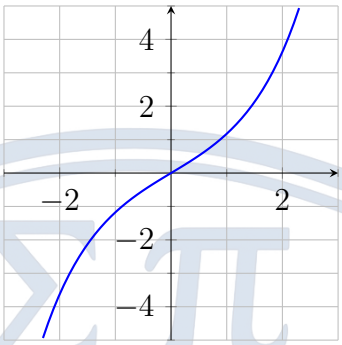
   (c) The graphs intersect at the origin $(0, 0)$ and at a second point P. Use your GDC to find the coordinates of P.

14. **[Paper 1 Style, Non-Calculator, Very Hard, 7 marks]**
Solve the following system of simultaneous logarithmic equations algebraically:

$$\log_2 x + \log_2 y = 4$$
$$\log_x y = 3$$

where $x > 0$, $y > 0$, and $x \neq 1$.

15. **[Paper 2 Style, Calculator Required, Very Hard, 8 marks]**
The hyperbolic sine function is defined as $f(x) = \frac{e^x - e^{-x}}{2}$ for $x \in \mathbb{R}$.



(a) Use your GDC to solve the equation $f(x) = 3$. Give your answer to three significant figures.

(b) Find an expression for the inverse function, $f^{-1}(x)$, algebraically. *(Hint: let $y = f(x)$ and multiply by $2e^x$ to form a quadratic equation in e^x).*