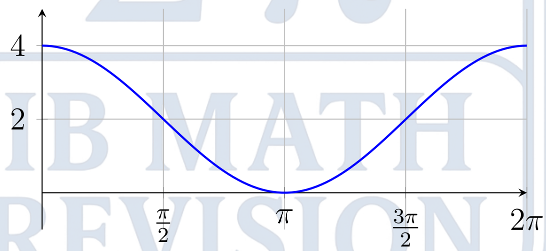
Unit 3: Trigonometric Functions & Graphs
IB Math AA SL

Answer all questions. Show all working. For Paper 1 questions, use analytical algebraic methods. For Paper 2 questions, use your graphic display calculator (GDC) efficiently.

1. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
The diagram below shows the graph of a circular function in the form $y = a\cos(bx) + d$, for $0 \le x \le 2\pi$.



By observing the maximum point, minimum point, and period of the graph, write down the values of the constants a, b, and d.

2. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
The height, h in metres, of a passenger on a Ferris wheel above the ground is modelled by the function:

$$h(t) = 20\sin\left(\frac{\pi}{30}t - \frac{\pi}{2}\right) + 25$$

where t is the time in seconds after the ride begins.

(a) Find the maximum height and the minimum height of the passenger above the ground.

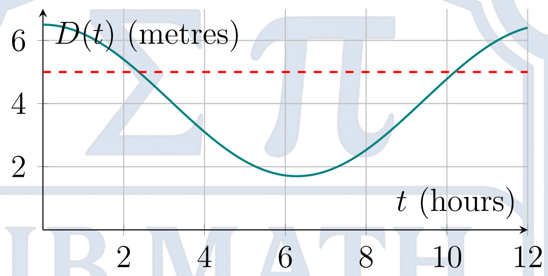
(b) Calculate the time it takes for the Ferris wheel to complete exactly one full revolution.

3. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
Consider the function $f(x) = \cos x$. The graph of $g(x)$ is obtained by applying the following sequence of transformations to the graph of $f(x)$:

- A horizontal stretch by a scale factor of $\frac{1}{2}$.
- A vertical stretch by a scale factor of 3.
- A reflection in the x-axis.
- A vertical translation upwards by 1 unit.

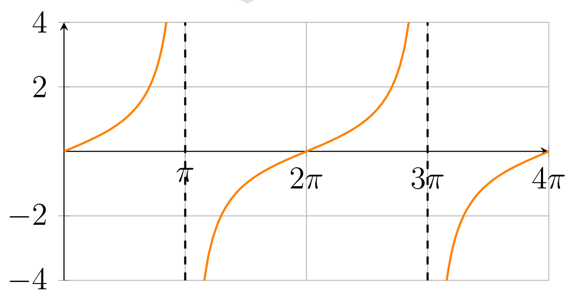
Write down an expression for $g(x)$.

4. **[Paper 2 Style, Calculator Required, Easy, 5 marks]**
The depth of water, D in metres, in a harbour at time t hours after midnight is given by $D(t) = 2.4\cos(0.5t) + 4.1$. The graph of the water depth is shown below.



A large ship can only enter the harbour when the depth of the water is greater than 5 m. The dashed line represents $D = 5$. Using your graphic display calculator, find the time interval (in hours after midnight) during which the ship can safely enter the harbour.

5. **[Paper 1 Style, Non-Calculator, Medium, 5 marks]**
The graph of $y = \tan\left(\frac{x}{2}\right)$ is shown below for $0 \leq x \leq 4\pi$.



(a) State the period of the function.

(b) Write down the exact equations of the two vertical asymptotes shown in the domain $0 \leq x \leq 4\pi$.

6. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
A patient's blood pressure, P in mmHg, at time t minutes is modelled by the periodic function $P(t) = 20\sin(140\pi t) + 100$.

   (a) Write down the maximum and minimum blood pressure of the patient.

   (b) Find the period of the function in minutes.

   (c) Hence, determine the patient's heart rate in beats per minute.

7. **[Paper 1 Style, Non-Calculator, Medium, 5 marks]**
A trigonometric function is of the form $f(x) = a\sin(bx) + d$, where $a, b > 0$. The graph of $f(x)$ has a local maximum at $\left(\frac{\pi}{4}, 5\right)$ and the very next local minimum is at $\left(\frac{3\pi}{4}, 1\right)$.

   (a) Find the exact values of the amplitude, a, and the principal axis, d.

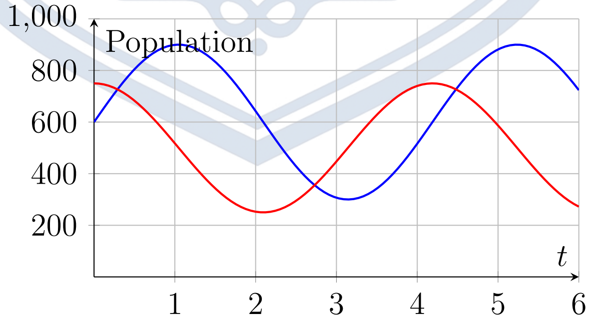
   (b) Determine the exact value of b.

8. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
The populations of two competing species of insects in a laboratory environment are modelled by the functions:

$$f(t) = 300\sin(1.5t) + 600$$

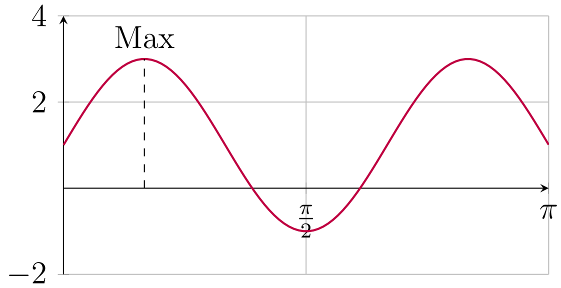
$$g(t) = 250\cos(1.5t) + 500$$

where t is the time in weeks since the experiment began. The graphs of $f(t)$ and $g(t)$ are shown below for $0 \leq t \leq 6$.



Use your graphic display calculator to find the exact times t during the first 6 weeks when the populations of the two species are exactly equal. Give your answers to three significant figures.

9. **[Paper 1 Style, Non-Calculator, Hard, 6 marks]**
The graph of $y = 2\cos(3(x - c)) + 1$ is shown below for $0 \leq x \leq \pi$. The constant c represents a positive phase shift where $0 < c < \frac{\pi}{2}$.



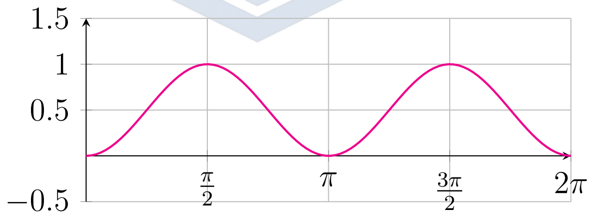
Given that the first maximum point occurs at $x = \frac{\pi}{6}$, deduce the exact value of c and find the exact coordinates of the y-intercept.

10. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
The ambient temperature T, in degrees Celsius, in a desert on a given day can be modelled by a cosine function in the form $T(t) = a\cos(b(t - c)) + d$, where t is the time in hours past midnight $(0 \leq t \leq 24)$. The minimum temperature of 14°C occurs at $04:00$ $(t = 4)$ and the maximum temperature of 28°C occurs at $16:00$ $(t = 16)$.

(a) Find the values of $a, b, c,$ and d, assuming $a > 0$ and $0 \leq c < 24$.

(b) Use your graphic display calculator to find the amount of time during the day when the temperature is strictly above 25°C. Give your answer in hours to two decimal places.

11. **[Paper 1 Style, Non-Calculator, Hard, 6 marks]**
The function $f(x) = \sin^2(x)$ is plotted below for $0 \leq x \leq 2\pi$.



(a) By using the double angle identity for cosine, show analytically that $f(x)$ can be written in the form $f(x) = A\cos(2x) + D$.

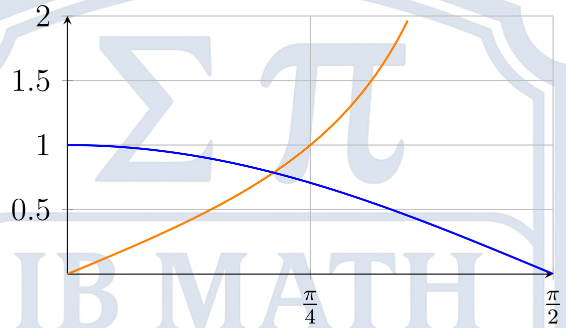
(b) Hence, write down the exact amplitude, period, and principal axis of the graph of $y = \sin^2(x)$.

12. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
The number of hours of daylight, S, in a northern city throughout the year is modelled by $S(t) = 3.5\sin\left(\frac{2\pi}{365}(t-80)\right) + 12$, where t is the number of days after January 1st ($1 \le t \le 365$).

    (a) Find the maximum number of hours of daylight, and the day of the year t on which it occurs.

    (b) A certain crop can only be successfully grown during the part of the year when there are strictly more than 14 hours of continuous daylight. Use your GDC to find the total number of consecutive days this crop can be grown.

13. **[Paper 1 Style, Non-Calculator, Very Hard, 7 marks]**
The graphs of $y = \tan x$ and $y = \cos x$ are shown intersecting in the domain $0 < x < \frac{\pi}{2}$.



    (a) By setting the equations equal to each other, show that the x-coordinate of the point of intersection satisfies the quadratic equation $\sin^2 x + \sin x - 1 = 0$.

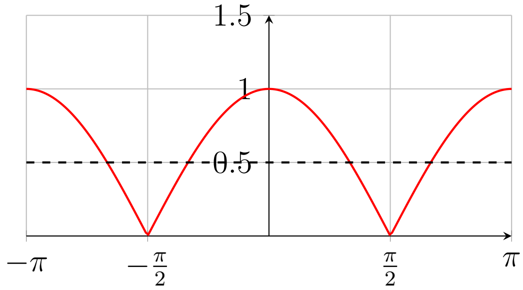
    (b) Using the quadratic formula, find the exact value of $\sin x$ at this intersection point.

14. **[Paper 2 Style, Calculator Required, Very Hard, 6 marks]**
A large Ferris wheel has a radius of 25 m and its centre is 30 m above the ground. It rotates at a constant speed, completing one full revolution every 40 seconds. A passenger boards the cabin at the very bottom of the wheel at $t = 0$.

    (a) Formulate a cosine model for the height $h(t)$ of the passenger above the ground at time t seconds.

    (b) Using your GDC, find the total amount of time during the first full revolution that the passenger is strictly higher than 40 m above the ground.

15. **[Paper 1 Style, Non-Calculator, Very Hard, 7 marks]**
The function $f(x) = |\cos x|$ is plotted below for $-\pi \leq x \leq \pi$.



(a) State the period of the function $f(x) = |\cos x|$.

(b) The dashed line represents $y = 0.5$. Solve the equation $|\cos x| = 0.5$ analytically to find the exact x-coordinates of all four intersection points shown in the graph.