IB MATHEMATICS AA HL
AHL TOPIC 5 PRACTICE

Implicit Differentiation, Related Rates & Inverse/Reciprocal Trig

Instructions to Candidates

- This practice paper contains **20** questions progressing from Easy to Very Hard.
- Each question indicates whether it is styled for **Paper 1 (No Calculator)** or **Paper 2 (Calculator Allowed)**.
- The paper tests syllabus topics AHL 5.14 and 5.15: Implicit differentiation, related rates of change, logarithmic differentiation, and the derivatives of $\sec x$, $\csc x$, $\cot x$, $\arcsin x$, $\arccos x$, $\arctan x$, a^x, and $\log_a x$.
- Answer all questions, showing all your working clearly.
- Total marks available: **96**.

Difficulty Progression

- **SECTION A (Easy):** Basic application of chain rule to inverse/reciprocal trig functions, finding dy/dx for simple implicit circles, and differentiating a^x and $\log_a x$.
- **SECTION B (Medium):** Implicit differentiation requiring the product rule, second derivatives of implicit functions, standard related rates problems (spheres, sliding ladders), and complex chain rule applications.
- **SECTION C (Hard):** Finding horizontal/vertical tangents on implicit curves, formal proofs of inverse trig derivatives, logarithmic differentiation (x^x), and complex 3D related rates involving similar triangles and trigonometry.

SECTION A: EASY (Fundamentals)

Question 1 (2 Marks) — Paper 1 (No Calculator Allowed)

Find the exact derivative of the function $f(x) = 5^x$ with respect to x.

Question 2 (3 Marks) — Paper 1 (No Calculator Allowed)

Given the function $y = \arctan(3x)$, find an expression for $\frac{dy}{dx}$.

Question 3 (3 Marks) — Paper 1 (No Calculator Allowed)

Find the derivative of $g(x) = \sec(2x)$ with respect to x.

Question 4 (3 Marks) — Paper 2 (Calculator Allowed)

Evaluate the derivative of $h(x) = \log_3(x)$ at the point where $x = 27$. Give your answer as an exact fraction.

Question 5 (4 Marks) — Paper 1 (No Calculator Allowed)

A circle has the equation $x^2 + y^2 = 25$. Use implicit differentiation to find the gradient of the curve at the point $(3, 4)$.

Question 6 (4 Marks) — Paper 2 (Calculator Allowed)

A circular oil spill is expanding such that its radius r is increasing at a constant rate of 2 cm s^{-1}. Find the rate at which the area of the oil spill is increasing when the radius is 10 cm.

Question 7 (3 Marks) — Paper 1 (No Calculator Allowed)

Differentiate $y = \arcsin\left(\frac{x}{2}\right)$ with respect to x, simplifying your answer to remove any complex fractions.

SECTION B: MEDIUM (Application & Algebraic Methods)

Question 8 (5 Marks) — Paper 1 (No Calculator Allowed)

A curve is defined implicitly by the equation $x^2y + y^3 = 8$. Find an expression for $\frac{dy}{dx}$ in terms of x and y.

Question 9 (5 Marks) — Paper 2 (Calculator Allowed)

A spherical balloon is being inflated such that its volume increases at a constant rate of $15\ \text{cm}^3\ \text{s}^{-1}$. Calculate the rate of increase of its radius at the instant when the volume of the balloon is $288\pi\ \text{cm}^3$.

Question 10 (5 Marks) — Paper 1 (No Calculator Allowed)

Consider the function $f(x) = e^{2x}\csc x$ for $0 < x < \pi$. Find $f'(x)$ and express your answer in a fully factorised form.

Question 11 (5 Marks) — Paper 1 (No Calculator Allowed)

Find the exact gradient of the tangent to the curve $y = \arccos(x^2)$ at the point where $x = \frac{1}{\sqrt{2}}$.

Question 12 (6 Marks) — Paper 2 (Calculator Allowed)

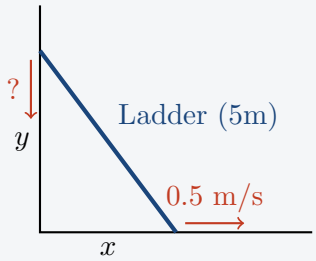
Given the function $y = x \cdot 3^x$, find the exact equation of the tangent line to the curve at $x = 1$. Give your answer in the form $y = mx + c$.

Question 13 (5 Marks) — Paper 1 (No Calculator Allowed)

A curve is defined by the implicit equation $xy = 4$. Using implicit differentiation to find $\frac{dy}{dx}$, show that the second derivative $\frac{d^2y}{dx^2}$ can be written in the form $\frac{ky}{x^2}$, where k is an integer to be found.

Question 14 (5 Marks) — Paper 2 (Calculator Allowed)

A ladder of length 5 m is leaning against a vertical wall. The bottom of the ladder is pulled away from the wall horizontally at a constant rate of 0.5 m s^{-1}.



Find the rate at which the top of the ladder is sliding down the wall at the exact instant the bottom of the ladder is 3 m from the wall.

SECTION C: HARD / VERY HARD (Synthesis & Proof)

Question 15 (6 Marks) — Paper 1 (No Calculator Allowed)

By rewriting $y = \arcsin x$ as $\sin y = x$, use implicit differentiation to formally prove that:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

Question 16 (6 Marks) — Paper 1 (No Calculator Allowed)

Use the method of logarithmic differentiation to find an expression for $\frac{dy}{dx}$ if $y = x^{\sin x}$, where $x > 0$.

Question 17 (7 Marks) — Paper 1 (No Calculator Allowed)

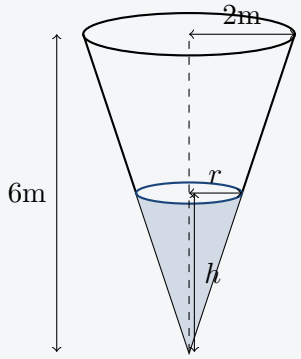
A curve is given by the implicit equation $x^2 - xy + y^2 = 3$.

(a) Find an expression for $\frac{dy}{dx}$. **[3 marks]**

(b) Hence, find the coordinates of the two points on the curve where the tangent is completely horizontal. **[4 marks]**

Question 18 (7 Marks) — Paper 2 (Calculator Allowed)

An inverted conical tank has a base radius of 2 m and a height of 6 m. Water is being pumped into the tank at a constant rate of 0.1 m^3 min^{-1}. Let h be the depth of the water and r be the radius of the water surface at time t.



Find the rate at which the depth of the water is increasing when the water is 3 m deep. Give your answer correct to 3 significant figures.

Question 19 (5 Marks) — Paper 1 (No Calculator Allowed)

Use the quotient rule and standard derivatives to find the exact derivative of $y = \frac{2^x}{\arccos x}$.

Question 20 (7 Marks) — Paper 2 (Calculator Allowed)

A tracking camera is situated 2 km away from a launch pad. A rocket is launched vertically upwards. When the rocket is exactly 3 km above the ground, its vertical velocity is 0.4 km s^{-1}. Calculate the exact rate of change of the camera's angle of elevation (θ) at this instant.