

pick out is swimming more agitatedly in the surrounding air, so that it does not strive as air. has to fall to the ground. We can but the weight of an air

determine the amount that is in a container by first weighing the container filled with air and then the air out pump and weigh it again. In the worst case the balance will show less. The difference is the weight of the air pumped out. One finds that one cbm (= 1 m<sup>3</sup>) of air weighs around 1.2 kg. If the weight of the room unit is denoted by  $\gamma$  (gamma), the barometer reading is for air at 15° and 760 mm

$$\gamma = 1.22 \text{ m/kg}^3.$$

By dividing it by the acceleration due to gravity  $g$ , we get the mass of the unit of space, also known as density. We denote the density with the letter  $\rho$  (Ro); it is accordingly

$\rho$

$\rho$

$$\frac{\gamma}{g} = \frac{1.22}{9.81} \frac{\text{kg}}{\text{m}^3} \frac{\text{sec}^2}{\text{m}}$$

=

=

approx.

8th

wed

A quantity of air of  $V$  cbm, which flows with the velocity  $v$ , has according to the above a working capacity of

$$A = V \cdot \frac{1}{16} v^2 \text{ mkg } ^1).$$

109

=

Every cbm of air that has a velocity of about 10 m/s could therefore with full exhaustion without air loss

16

do around 6 mkg of work.

If the speed is only 5 m/s, the working capacity is:

$$\frac{5^2}{16} = \text{rd. } 1\frac{1}{2} \text{ mkg. } ^1$$

### **b) energy extraction.**

The above considerations show how the work capacity of moving air can be determined. We must now turn to the question of how to remove this work from the air. In itself, there is practically any amount of air and thus any amount of work capacity. The next question is therefore how much of this air volume can be captured and forced to work.

At

removal

Let's imagine a pipe with a clear cross-section  $F$ , placed in the wind in such a way that the air can flow unhindered through the pipe Energy(Fig. 3). If  $v$  again means the wind speed, then in every second a quantity of air  $v \cdot F$  enter the

<sup>1</sup> 1) The values given for  $\gamma$  and  $e$  only apply to normal barometric reading and normal temperature. On high mountains, where the barometer reading is significantly lower, the correspondingly lower density of the air must also be taken into account in calculations.

tube on one side and exit just as much at the other end. But if a device is housed inside the tube, which of the

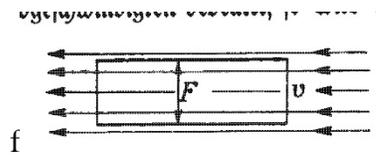


Fig. 3

flowing air - we want to postpone the question of what such a device looks like for the time being, so that's just up

#### Wind energy and its use by windmills

Cost of speed possible. The air will therefore exit at a lower velocity than it had when it entered. On the other hand, however, the same amount of air must exit as enter, since otherwise an accumulation (compression) of the air or a dilution of the same would occur, which of course is unthinkable in the long run. We can take this situation into account by suitably widening the tube (Fig. 4). If the entrance cross-section  $F_1$  and the entrance velocity  $v_1$  and

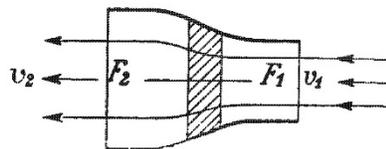


Abb. 4

be great that

Fig. 4

$$F_2 v_2$$

==

If we reduce this speed in the tube by energy extraction to the amount  $v_a$  (the shaded area in Fig. 4 may indicate the energy-removing device), then the outlet cross-section  $F$  must be as follows

$F_1 V_1 =$  flow rate per second. If we did not meet this condition, the air would no longer flow undisturbed into the inlet opening. Fig. 5 shows the process roughly as it would occur in an unexpanded pipe when energy is removed. The incoming air only partly flows into the tube, the rest of it deviates to the side and flows outside

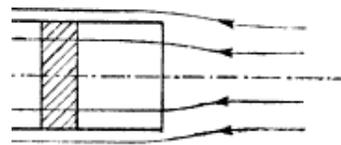


Abb. 5

Rohr entlang. Die Durchflussmenge richtet sich in solchen und ähnlichen Fällen stets nach dem Austrittsquerschnitt  $F_2$  und der Austrittsgeschwindigkeit  $v_2$ , sie ist

$$Q = F_2 v_2.$$

The more 1 cbm air of the speed  $v$ . has, as we saw above, a

Energy we work capacity

the through

flowing

air

To move

$$A_1 = \frac{Q}{v_1}.$$

for the inner If we reduce the speed to the amount  $v_e$ , then the work will only be able to

exit

speed

thing.

$$A_2 = \frac{Q}{2} v_2^2$$

If we disregard energy losses, we can therefore do a job

e

$A_1 - A_2 = 2(v_1 - v_2)$  win. Now if we every second

Q cbm process in this way, so we can do a work in every second

$$L = Q (v_1 - v_2)$$

win. Such work per second is also called power, the unit is mkg/sec.

Instead of this last unit, other units are very often used in practice. The most important are: the horsepower (hp) and the kilowatt (KW). 75 mkg/sec gives 1 hp and 102 mkg/sec gives approximately 1 KW (compare the summary of the most important units, Appendix Table 2).

The more energy we extract from an amount of air, the smaller the exit velocity  $v_e$ . Therefore, if we have a certain tubular structure with a given exit cross-section, the less air will flow through it, the more energy we extract from each cbm of air. For example, would we If, for example, the air flowing through would take away all its energy, almost nothing would flow through our tube because of the infinitesimally small exit velocity, but almost everything

would pass by the outside. As a result, we would no longer be able to get any significant performance out of it. On the other hand, if we didn't extract any energy from the air flowing through, the air would flow through with undiminished speed, i.e. in large quantities, but we would still not gain any energy, since we are not extracting any energy from the air.

wind  
wheels.

This consideration was mainly intended to show that the power, form and work we can extract from the wind at a given wind speed is limited by the size of the structure, and that we can only extract a normal part of the energy from the air if we are of a given size of the building want to gain the greatest possible performance. We now want to take a closer look at real wind turbines and apply the considerations just made to them. The typical shape, which recurs in detail in most windmills with various modifications, is shown in Fig. 6. A number of blades (blades) are attached to a wave that is in the direction of the wind, which are at an angle to the direction of the wind. The wind pressure  $R$  on these blades is also directed obliquely due to the inclination of the same and therefore has a component  $T$ , which causes the wheel to rotate and thereby performs work. We will come back to these processes on the wings in more detail later. First of all, we only want to be interested in it to the extent that we can get an idea of what the above-mentioned device for extracting energy from the wind looks like.

Windmill.

In contrast to our previous consideration, it is striking here that there is no pipe enclosing the windmill. But one

can easily see that such a tube is not necessary at all, since the air fills out those cross-sections of its own accord which it needs at the remaining speed. But since there is no pipe, we are missing the exit cross-section, which played an important role in the last considerations. Instead, we have the surface of the pinwheel here.

The essential question that we now have to ask ourselves is therefore, first of all, in an idealized way: how much air flows through a wind turbine of a given diameter per second? and further: how much energy can we extract from it in the best case? If, in the treatment of these questions, a little more mathematical formulas appear in the following than maybe

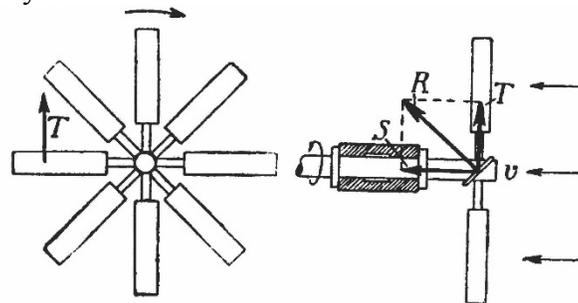


Abb. 6. Kräfte am Windrad.

Fig. 6. Forces on the wind turbine.

Some readers are accustomed to reading these uncomfortable passages and confining themselves to the rest of the text, in which the results of the calculations are explained as far as is necessary to understand the train of thought. In order not to complicate the treatment of the above fundamental questions unnecessarily by unessential secondary circumstances, we want to idealize the windmill as much as possible: We want to situate the windmill through a circular, air-permeable disc and assume that it is possible for us at every point of this disc to to withdraw as much energy from the air flowing through as is currently

favorable. One may ask whether such an idealization does not go too far, so that the results of the consideration lose their practical meaning. With a 4-bladed or even a 2-bladed wind turbine  $z$ . For example, at first glance it seems inappropriate to arbitrarily allow energy to be extracted over the entire circular area swept by the wings. We will come back to these special conditions later and find that the deviations from our assumptions above are only insignificant. The inner reason for this initially striking appearance is that the wings

Although at a certain point in time they only exert forces on relatively small areas of the air, they affect all parts of the circle one after the other as they go around. Such an idealized windmill may be represented by the dashed line in Fig. 7 above. The wind arrives with the speed  $v_i$ , so much energy is withdrawn from it in the wind wheel that the speed behind the wheel is only  $v$ . amounts.

The transition from one speed to the other cannot take place abruptly, since this also involves an increase in cross-section and, in the event of a sudden transition, the streamlines would no longer connect to one another. The Vor

$$v_2 \leftarrow S \leftarrow$$

P

c

gang is such that the air already builds up a little in front of the wind turbine. Their speed is converted into pressure, similar to that of a body falling onto a resilient base (f. the explanation of kinetic energy above), where the kinetic energy is converted into the working capacity of the spring. As a result, the air arrives at the wind turbine at a

reduced speed but with increased pressure. The energy extraction in the Fig. 7. Above flow through an idealized wind wheel now first causes the wind wheel (the dashed line should only indicate a reduction in the wind wheel). Below course of the speed ( $v$ ) pressure energy, so that the air, and the pressure ( $p$ ) in front of and behind the wind turbine. which arrived in front of the wheel with increased pressure and continues to flow behind the wheel with reduced pressure. The speed itself cannot change at all within the imaginary infinitely thin wheel. Only behind the wheel does the speed reduction continue again, in that the speed energy is converted into pressure until the normal air pressure is restored (compare Fig. 7 below).

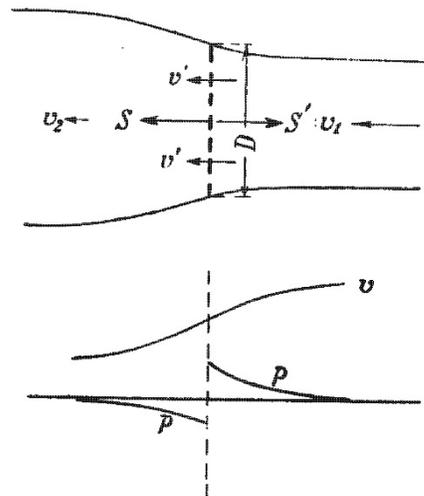


Abb. 7. Oben Strömung durch ein idealisiertes Windrad (die gestrichelte Linie soll das Windrad andeuten). Unten Verlauf der Geschwindigkeit ( $v$ ) und des Druckes ( $p$ ) vor und hinter dem Windrad.

Accordingly, the air does not flow through the wind turbine with the initial velocity  $v$ . still with the final velocity  $v'$ , but with an intermediate velocity  $v'$ . This circumstance complicates our reasoning somewhat. If we want to determine the energy that we can extract from the air with the wind wheel, we must first know how much air actually passes through the wind wheel, and for this we need the flow rate  $v'$ , because the flow rate per second

the wind

$D^2 \pi / 4$

quantity is  $Q = Fv'$ , where  $F$  is the circular area of the wind turbine,

$D$  should be the diameter of the wind wheel.

Fortunately, one can show that the velocity  $v'$  is just speed is the mean of the initial and final speeds

thing in

pinwheel

is this

arithmetic

$$v' = \frac{v_1 + v_2}{2}$$

Means from To this end, we first want to consider the magnitude of the force  $S$ , the wind which the wind exerts on the wheel. If a mass  $m$  is moved by a certain amount of velocity  $v_1 - v$ . If you want to slow things down, you have to apply a force to the mass for one second

behind the windmill.

$S'm (V_1 - V_2)$

exercise in the opposite direction to the movement. If the wind turbine experiences a force  $S$  in the direction of the wind, it exerts an equal force  $S'$  in the opposite direction on the air. The force  $S'$  causes the wind speed to be decelerated

from the amount  $v_1$  to the amount  $v_2$ . Now a mass flows in one second

$$m = \rho v F$$

through the wheel. The mass is subjected to the decelerating effect of the force  $S'$  during the flow, ie for one second. Therefore, if this mass is thereby decelerated from the speed  $v_1$  to the speed  $v_2$ , the force must

$$S' m (v_1 - v_2) = \rho v F (v_1 - v_2)$$

be. The force  $S$  exerted by the wind on the wind wheel is just as great, just sitting in the opposite direction.

During this process, the energy of the air volume flowing through the wheel every second drops from the amount  $V_1$  to the amount

$m$

$2$

$m$

$$v_2^2$$

$2$

Die Differenz

$$L = \frac{m}{2} (v_1^2 - v_2^2)$$

1) Here it is tacitly assumed that no forces other than those originating from the wind wheel are exerted on the air flowing through. In the present case, this also applies to the entire air flowing through the wind turbine. It would apply e.g. E.g. not possible if different pressure prevails in front of and behind the wheel, as is the case with turbines in closed pipes. Furthermore, it does not apply to each

individual current filament that penetrates the wind turbine, since the individual current filaments exert centrifugal forces on one another as a result of  $V_1+V_2$  the curvature of the current paths.  $v' = 2$

therefore only represents the mean value for the flow rate. At individual points on the wheel,  $v'$  is greater than this mean value and at others it is smaller. In the following, however, due to a lack of more precise knowledge, we will often assume that the flow velocity at each point is the arithmetic mean of the initial and final velocity

is delivered to the wind turbine every second and, if no losses occur, could be recovered as nut energy. Let's not look at the situation far in front of and behind the wheel, as we just did, but in the immediate vicinity of it. Here the air flows through the wheel with the speed  $v'$  and has to overcome the force  $S'$ . We can imagine the process as if we push a closed volume of air through the wheel with the speed  $v'$  (Fig. 8). The overpressure in front of the wheel and the under

The pressure behind it causes the force  $S$  on the wheel. However, they also act on the end walls in the same way, so that a force  $S$  is also required to move them. Therefore, if one pushes the air through the wheel in the manner indicated, one has to do the work per second

$$L = Sv'$$

$v'$ .

$v'$

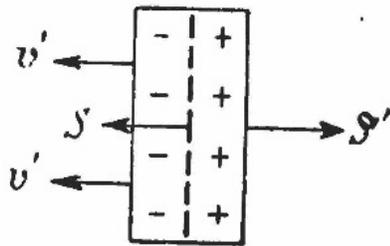


Abb. 8. Arbeitsleistung beim Durchtritt der Luft durch das idealisierte Windrad.

Fig. 8. Work performance at

Passage of the air through the idealized wind wheel.

afford, which is transferred to the wind turbine. In reality, the work is not done by moving the walls, but is taken from the kinetic energy of the air. But that doesn't matter for the amount of the service.

We can therefore derive the power given off by the air from the energy of the air (see above)

$$L = \frac{m}{2} (v_1^2 - \dots)$$

on the other hand, however, also from the thrust force  $S$  of the wind wheel and the flow rate  $v'$

$$L = Sv'$$

Since the same thing must come out in both ways, it must

$$Sv' = \frac{m}{2} (v_1^2 - v_2^2)$$

fein. Da aber, wie wir sahen

$$S = m (v_1 - v_2)$$

ist, so ist

$$m (v_1 - v_2) v' = \frac{m}{2} (v_1^2 - v_2^2),$$

2

=

2

and since  $(v_1^2 - v_2^2) = (v_1 - v_2)(v_1 + v_2)$ , it follows that

$$v' = \frac{1}{2} (v_1 + v_2)$$

wie wir es bereits oben angaben.

With this knowledge we can relatively easily calculate the work which is taken from the wind. The second through that

Bes, wind energy.

2

Größte Rad strömende Masse ist  
Energie-  
menge,  
welche ein  
Rad von  
gegebenem  
Durch-  
messer

$$m = \rho v F = \frac{\rho}{2} (v_1 + v_2) F.$$

Damit wird

$$L = \frac{m}{2} (v_1^2 - v_2^2) = \frac{\rho}{4} (v_1^2 - v_2^2) (v_1 + v_2) F.$$

one

winds of

In order to have a suitable standard of comparison for this power, let us, given it, compare it with that power which is available in a quantity of air speed

energy that can pass through an equally large cross-section F every second.

$$D^2 \pi$$

=

flows,

4

when no work is done, so that the air flows through with its full velocity  $v_1$ . This achievement is

$$L_0 = \frac{\rho}{2} v_1^3 \cdot F.$$

Damit wird

$$\frac{L}{L_0} = \frac{1}{2} \left[ 1 - \left( \frac{v_2}{v_1} \right)^2 \right] \left[ 1 + \frac{v_2}{v_1} \right]$$

$L_0$  depends only on the wind speed  $v_1$  and the diameter or of the area F of the wind wheel. The power L, on the other hand, also depends on the ratio of the wind speeds in front of and behind the wheel.

$v_2$

$v_1$

$v_2$

We must now ask ourselves the question at what ratio of  $v_2/v_1$ , all other things being equal, can we gain the most energy (ie  $L/L_0$  is greatest) and how much energy then results. If we calculate according to the last equation for different values of and plot the result in a diagram, we get Fig. 9).

L

$L_0$

L

$L_0$

$v_2$

We find the largest value of  $16/27 \approx$  (almost equal) 0.6

namely, this maximum value results when

$v_2$

$v_1$

$1/3$  is. The largest

The power that we can extract from the wind with a wind turbine with a diameter of  $D_m$  at a wind speed of  $v_m/s$  is therefore

$$\left| L_{\max} = \frac{16}{27} \cdot \frac{\rho}{2} v^3 \cdot \frac{D^3 \pi}{4} \text{ mkg/sek} \right|$$

1) The considerations applied here are no longer permissible for very small values of  $v_2/v_1$ , since the influences indicated in note 1 p. 10 then play a too important role.

oder da  $\frac{\rho}{2} \approx \frac{1}{16}$  ist, wird

$$\begin{aligned} L_{\max} &= \frac{v^3}{27} \cdot \frac{D^3 \pi}{4} \text{ mkg/sek} \\ &= 0,000388 v^3 D^3 \text{ PS} \\ &= 0,000285 v^3 D^3 \text{ KW}. \end{aligned}$$

The outputs resulting from this formula for various wind speeds and wheel diameters are summarized in Table 3 and Diagram 10 (Appendix p. 56 and 59).

It is often surprising that the power that is extracted from the wind does not always increase, the more wing surface is accommodated in the wind wheel and the more the wind speed is slowed down with it. The

0.6

0.4

4/Lo 10:2

0.2

0.4

0.6

0.8

1.0

igr

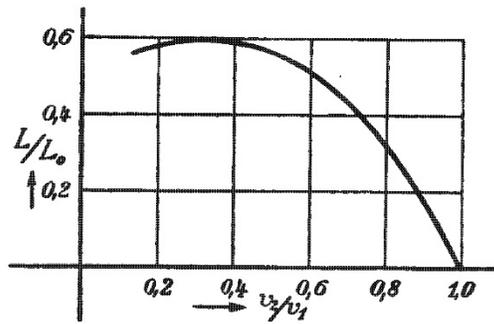


Abb. 9. Abhängigkeit der theoretischen Winddrableistung von der Stärke der Abbremsung des Windes.

Fig. 9. Dependence of the theoretical wind turbine output on the strength of the deceleration of the wind.

The reason is that when the blades are too close together, only a small part of the air goes through the wind wheel, while the rest of the air escapes the wheel and flows around the outside.

Due to ignorance of these conditions, after the Flettner rotors became known, a large number of inventors made the following incorrect conclusion. The conclusion of rotors give considerably greater forces than wings of the same size. As a result, I get much more power when using rotors on the windmill blades. In reality, however, if the rotors are not made small in diameter in accordance with their greater effectiveness, they would, like oversized blades, slow down the wind considerably, so that less air would pass through the wheel. The consequence of this is that not more, but less energy is gained.

Since the conversion of wind energy into mechanical energy does not take place without losses and, in addition, the reduction in speed  $V_a$  does not always correspond to the most favorable conditions, the

$V_1$

actual output  $L_2$  of a wind turbine is always smaller than the  $L_{max}$  calculated above. The ratio of real nut performance to theo

1) f. Note p. 5.

rotating cylinders instead of wings.

retic maximum can be referred to as the efficiency  $\eta$  of the wind turbine.

$$\eta = \frac{L_n}{L_{\max}}$$

$$L_n = \eta \cdot L_{\max} = \eta \cdot \frac{16}{27} \frac{\rho}{2} v^3 \cdot \frac{D^2 \pi}{4}$$

Zur Darstellung von Versuchsergebnissen mit Windrädern bildet man vielfach das Verhältnis  $\frac{L_n}{L_o} = \frac{L_n}{\frac{\rho}{2} v^3 \frac{D^2 \pi}{4}}$  und bezeichnet es als Leistungsziffer  $c_l$

normal

(compare Fig. 32). Accordingly, there is a relationship between this coefficient of performance and the degree of efficiency  $\eta$

$$c_l = \frac{16}{27} \eta.$$

Since the ratio for the dimensioning of the wind turbines is an exception

$L_{\max}$

$L_o$

It is appropriate to consider the extent to which the value 16/27 found for enjoyment depends on special assumptions and whether it is possible to increase it. One can easily consider that by arranging a second wind turbine behind the first one, the energy that is still present in the wind there can only be partly used for the. Our derivation is therefore only valid for a single, slightly rounded, almost disc-shaped wind turbine. If you are free to use the space behind the installed wind turbine to generate energy as you wish, but without exceeding the specified  $L_{\max}$  diameter, you can do 1. The Wind $L_o$  mill will then take the form of a very long cylinder. The interior of this cylinder does not need to contain any energy-draining organs. It is sufficient if the cylinder surface and the rear floor are suitable for this (Fig. 11, the dashed line represents the energy-draining organs). However, such an arrangement is unlikely to be of any practical importance, since the great

length (l) of such a wind turbine naturally increases the manufacturing costs in

wind wheel shape.

They are orders too conceivable, at

those that

Out of

D

nußungs- Fig. 11. Pinwheel shape with increased theoretically propagated in a similar manner, as a

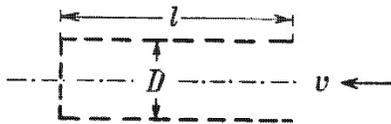


Abb. 11. Windradform mit erhöhter theoretischer Leistung.

ratio the value 1

reached, or

even over

Performance.

larger diameter.

One can even think of arrangements where  $L_{max}$  is greater than 2

$$v^3 \cdot \frac{D^2 \pi}{4}$$

strides. is. Fig. 12 shows such an arrangement: Two wheels are on a common shaft