POWER TRANSFORMERS

Voltage Calculation in the Terminals

INDEX **PAG.**

1 - PURPOSE

The purpose of this technical information is to analyze the operating conditions of power transformers, which feed a load or set of loads, to calculate the voltage drop in the terminals, during permanent or transient operation, and to develop Excel spreadsheets to perform the calculations.

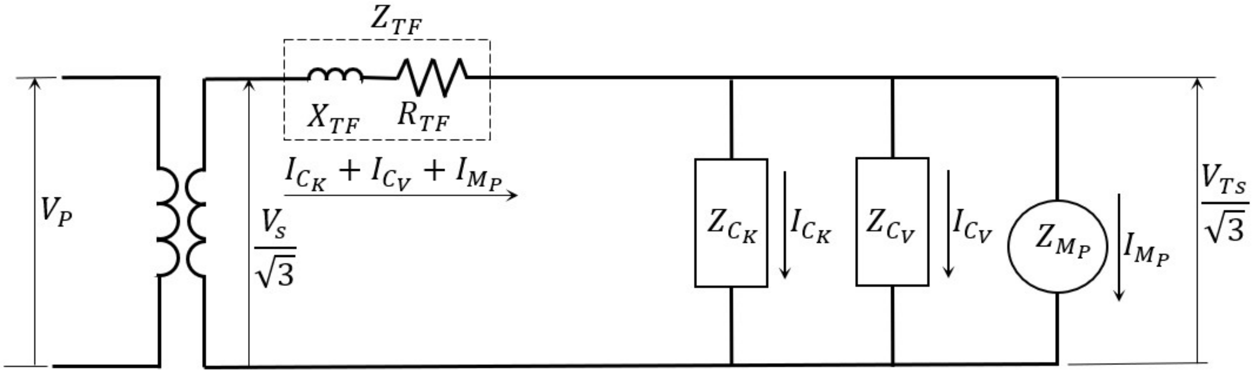
2 - REFERENCE DOCUMENTS

2.1 - Spreadsheets

PL.EL.SA.CA.01.R1 Power Transformers - Voltage Calculation in the Terminals

3 - BASIC CIRCUIT

The figure below represents the basic secondary circuit of a three-phase transformer, connected in delta in the primary and star, solidly grounded, in the secondary, fed the various types of loads that are normally found in practice. Loads can be constant power, variable power, and start of motors.



Where:

V_P	Primary voltage in transformer (V)
V_s	Secondary voltage in transformer (V)
V_{Ts}	Voltage secondary of transformer (V)
I_{C_K}	Current of the constant load (A)
I_{C_V}	Current of the variable load (A)
I_{M_P}	Motors current at starting (A)
Z_{TF}	Transformer impedance (Ω)
R_{TF}	Transformer resistance (Ω)
X_{TF}	Transformer reactance (Ω)
Z_{C_K}	Impedance of the constant load (Ω)
Z_{C_V}	Impedance of the variable load (Ω)
Z_{M_P}	Impedance of the motor(s) at start (Ω)

4 - TRANSFORMERS

Transformers are usually with two windings, provided with taps in the primary winding to compensate for voltage variations in the power supply. These taps are ± 2 x 2.5% for transformers with manual tap changer, without load, or ± 8 x 1.25% for transformers with automatic load tap changer.

4.1 - Voltages

The secondary voltage in the transformer depends on the voltage applied in the primary and the tap used. Thus, if transformer ratio is V_{P_n}- V_{S_n} with taps in the primary, the secondary voltage V_s in the transformer will be:

$$V_S = \frac{V_P}{k\,\frac{V_{P_n}}{V_{S_n}}}$$

That is.

$$V_S = \frac{V_P V_{S_n}}{k\, V_{P_n}}$$

Where:

V_s- Secondary voltage in the transformer (V)

V_P- Primary voltage in the transformer (V)

V_{P_n}- Primary rated voltage of transformer (V)

V_{S_n}- Secondary rated voltage of transformer (V)

k- Used tap of the secondary winding of step-up transformer in pu. For example, for the tap -5% k=0.95; for the tap -2.5% k=0.975; for rated tap k=1; for the tap +2.5% k=1.025; for the tap -5% k=0.95.

$$k = \frac{V_{P_x}}{V_{P_n}}$$

Where:

V_{P_x}- Primary voltage on tap x in the transformer (V)

4.2 - Impedance

The transformers impedance is obtained in the performance of the factory tests, when the values of impedance, resistance and, consequently, reactance are determined. However, unless the test reports are available, only the percentage impedance of the transformers will be available, this impedance must be indicates on the transformers data plate.

4.3 - Resistance and Reactance

As the values of impedance, resistance and reactance are only known after the tests of transformers, it is necessary to adopt values to use in the calculations and elaboration of the projects. The available value is, usually, the rated impedance, which is defined in standards and by manufacturers, or by the user, in special cases.

Impedance is composed of reactance, which is a fixed value, and resistance, that varies depending on temperature. These values are not defined in the standards or by manufacturers. However, in the absence of these data we can consider the literature on the subject, where we find simulations that, for example, consider the following alternatives:

In the First Edition of the Industrial Power Systems Handbook (Donald Beeman), for a transformer of 1500kVA, whose rated reactance is 5½%, in the calculation of voltage drop is made the consideration that:

$Z = 1 + j5.5$

This implies that $Z = 5.590\%$

In the First Edition of the Manual of Low Voltage (Volume 1), from Siemens, for a transformer of 1600kVA, whose rated impedance is 6%, in the calculation of voltage drop is made the consideration that:

$6 = 1 + jX$

This implies that $X = 5.916\%$

Considering that the two cases refer to impedances, the difference is that one considers the data of the transformer reactance and the other the impedance, but both consider the resistance with a value of 1% and the result will be the same. However, as in Brazil, the information contained in the transformers data plate is the impedance, in this informative, the rated impedance of the transformer will be considered as input data for the calculations.

The rated impedance of transformers, still in the manufacturing phase, may change depending on the tolerances allowed by the applicable standards. For this reason, the impedance of the transformers is only engraved on the data plate after the tests have been carried out at the factory. The impedance indicated on the transformer data plate is referred to a power and a temperature, which depends on the insulation class used.

4.4 - Short Circuit Voltage

The transformer impedance is also called short-circuit voltage because, with the short-circuited low voltage winding, it corresponds to the percentage ratio between the voltage applied to the terminals of the primary winding, which circulates the rated current in the secondary winding, and the rated voltage of the primary winding. For example, if the impedance of a 13800/480V ratio transformer is 6%, the voltage that must be applied to the 13800V winding, to circulate the rated current in the 480V circuited short winding, must be 6% of 13800V, i.e., 828V.

4.5 - Rated Impedance

The impedance of the transformer also depends on the used tap of the winding in the primary and, considering that for any tap used, the power of the transformer is constant, and equal to the rated power (P_{TF_n}), we have:

Rated impedance:

$$Z_{TF_n} = \frac{{V_{P_n}}^2}{P_{TF_n}} . z_n$$

$$P_{TF_n} = \frac{{V_{P_n}}^2}{Z_{TF_n}} . \frac{z_n}{100}$$

Where:

Z_{TF_n}- Rated transformer impedance (Ω)

V_{P_n}- Rated primary voltage of transformer (V)

P_{TF_n}- Rated power of transformer (VA)

z_n- Rated impedance of transformer (%)

4.6 - Impedance on Tap

Impedance for any x tap:

$$Z_{TF_x} = \frac{{V_{P_x}}^2}{P_{TF_n}} \cdot \frac{z_n}{100}$$

Where:

Z_{TF_x}- Transformer impedance in tap x (Ω)

V_{P_x}- Rated primary voltage of tap x of transformer (V)

P_{TF_n}- Rated power of transformer (VA)

z_n- Rated impedance of transformer (%)

Replacing P_{TF_n}:

$$Z_{TF_x} = \frac{{V_{P_x}}^2}{\frac{{V_{P_n}}^2}{Z_{TF_n}} \cdot \frac{z_n}{100}} \cdot \frac{z_n}{100}$$

$$Z_{TF_x} = Z_{TF_n} \frac{{V_{P_x}}^2}{{V_{P_n}}^2}$$

Replacing V_{P_x} by:

$$V_{P_x} = kV_{P_n}$$

And Z_{TF_x} by:

$$Z_{TF_n} = \frac{{V_{P_n}}^2}{P_{TF_n}} \cdot \frac{z_n}{100}$$

We have:

$$Z_{TF_x} = \frac{{V_{P_n}}^2}{P_{TF_n}} \cdot \frac{z_n}{100} \frac{(kV_{P_n})^2}{{V_{P_n}}^2}$$

$$Z_{TF_x} = \frac{{V_{P_n}}^2}{P_{TF_n}} \cdot \frac{z_n}{100} \frac{k^2{V_{P_n}}^2}{{V_{P_n}}^2}$$

$$Z_{TF_x} = \frac{k^2{V_{P_n}}^2}{P_{TF_n}} \cdot \frac{z_n}{100}$$

Considering $Z_{TF_x} = Z_{TF}$ the impedance of the transformer for any tap x:

$$Z_{TF_x} = \frac{k^2{V_{P_n}}^2}{P_{TF_n}} \cdot \frac{z_n}{100}$$

The impedance on the low voltage side will be:

$$Z_{TF} = \frac{k^2{V_{P_n}}^2}{P_{TF_n}} \cdot \frac{z_n}{100} \cdot \frac{{V_{S_n}}^2}{{V_{P_n}}^2}$$

$$Z_{TF} = \frac{k^2{V_{S_n}}^2}{P_{TF_n}} \cdot \frac{z_n}{100}$$

Where:

k- Used tap of the secondary winding of step-up transformer in pu. For example, for the tap -5% k=0.95; for the tap -2.5% k=0.975; for rated tap k=1; for the tap +2.5% k=1.025; for the tap -5% k=0.95.

V_{S_n}- Rated secondary voltage of transformer (V)

P_{TF_n}- Rated power of transformer (VA)

z_n- Rated impedance of transformer (%)

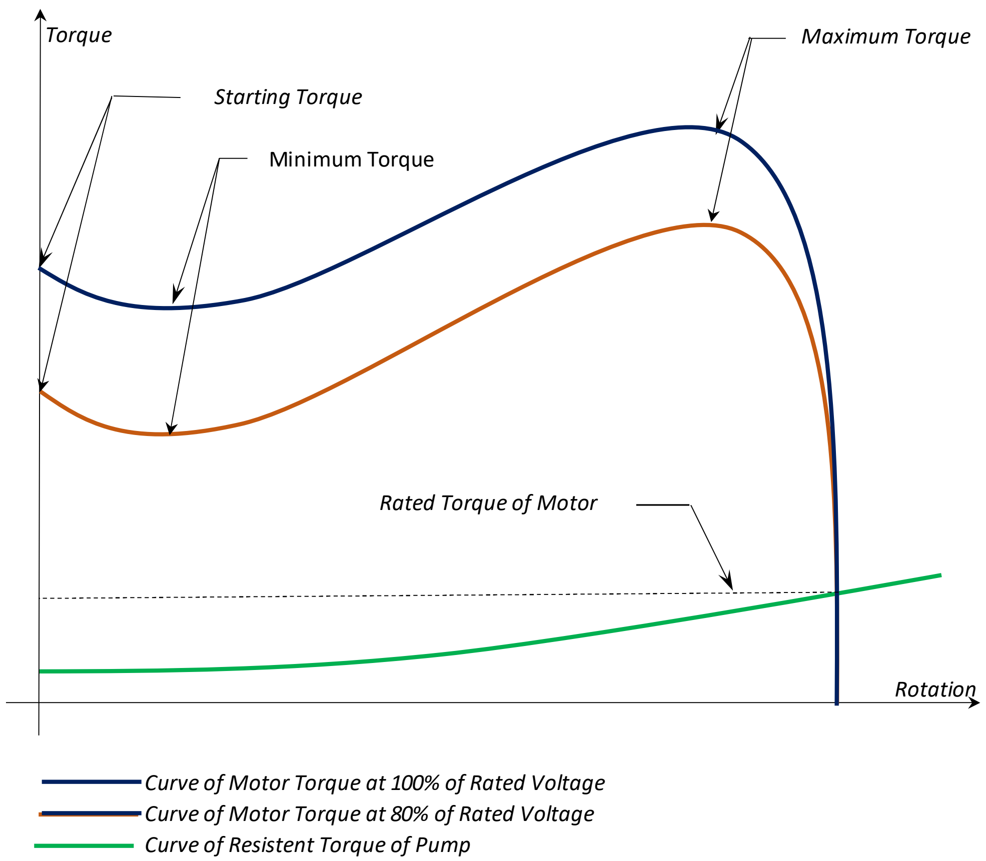
4.7 - Impedance Angle

Whereas:

$$\boldsymbol{\theta_{Z_{TF}} = arc\ cos(\frac{R_n}{z_n})}$$

5 - CONSTANT POWER LOADS

In constant power loads, the current varies depending on the voltage to maintain constant power. In this case are, by example, battery chargers, communication systems and, mainly, induction motors. Induction motors have the characteristic of keep, practically, constant the rotation with the voltage variation (see figure below).



$Power = Force\ x\ Velocity$ or $Power = Torque\ x\ Angular\ Velocity$

Note that the motor torque varies during motor start and is different for each voltage value, but, during operation, the torque and rotation remain constant. Therefore, as the load (Force or Torque) and speed remain constant, the Power also remains constant, that is, the current variation is inversely proportional to the variation of the voltage.

5.1 - Power of Constant Load

The power of the constant load in the circuit is given by:

$$P_{C_K} = \frac{{V_{Ts}}^2}{Z_{C_K}}$$

Where:

P_{C_K}- Constant load power (VA)

V_{Ts}- Voltage in the secondary terminals of transformer (V)

Z_{C_K}– Impedance of a constant load (Ω)

At constant load the power is constant, and equal to the rated power of the load, i.e.:

$$P_{C_{Kn}} = \frac{{V_{C_{Kn}}}^2}{Z_{C_{Kn}}}$$

$P_{C_{Kn}}$- Rated power of constant load (VA)

$V_{C_{Kn}}$ - Rated voltage of constant load (V)

$Z_{C_{Kn}}$– Rated impedance of constant load (Ω)

5.2 - Impedance of Constant Load

As the load power is constant:

$$Z_{C_K} = \frac{{V_{Ts}}^2}{P_{C_{Kn}}}$$

5.3 - Impedance Angle of Constant Load

Because the load power factor is, normally, an arbitrated value, for example, equal to 0.85,

$$\theta_{C_K} = arc\ cos\left(FP_{C_K}\right)$$

Where:

FP_{C_K} Power factor of constant load

5.4 - Current of Constant Load

How:

$$\overrightarrow{I_{C_K}} = \frac{\frac{\overrightarrow{V_{Ts}}}{\sqrt{3}}}{\overrightarrow{Z_{C_K}}}$$

$$\overrightarrow{I_{C_K}} = \frac{(\frac{V_{Ts}}{\sqrt{3}}, 0)}{(Z_{C_K}, \theta_{C_K})}$$

$$\overrightarrow{I_{C_K}} = \left(\frac{V_{Ts}}{\sqrt{3}Z_{C_K}}, -\theta_{C_K}\right)$$

But how,

$$Z_{C_K} = \frac{{V_{Ts}}^2}{P_{C_{Kn}}}$$

$$\overrightarrow{I_{C_K}} = \left(\frac{P_{C_{Kn}}}{\sqrt{3}V_{Ts}}, -\theta_{C_K}\right)$$

6 - VARIABLE POWER LOADS

In variable power loads impedance is a constant value. Therefore, the variation of the voltage causes the current to vary as a function of the impedance of the load. In this case we can consider loads composed of transformers, reactors and resistors. In these loads the current is directly proportional to the voltage variation.

6.1 - Variable Load Power

In variable power load we have:

$$P_{C_V} = \frac{{V_{Ts}}^2}{Z_{C_V}}$$

Where:

P_{C_V} Power of variable load (VA)

V_{Ts} Secondary voltage on transformer terminals (V)

Z_{C_V} Impedance of variable load (Ω)

The rated power of the variable load and:

$$P_{C_{Vn}} = \frac{{V_{C_{Vn}}}^2}{Z_{C_{Vn}}}$$

Where:

$P_{C_{Vn}}$ Rated power of variable load (VA)

$V_{C_{Vn}}$ Rated voltage of variable load (V)

$Z_{C_{Vn}}$ Rated impedance of variable load (Ω)

6.2 - Variable Load Impedance

It happens that the load is variable because the impedance is constant (for example, resistor), so:

$$Z_{C_V} = Z_{C_{Vn}}$$

$$Z_{C_V} = Z_{C_{Vn}} = \frac{{V_{C_{Vn}}}^2}{P_{C_{Vn}}}$$

$$Z_{C_V} = \frac{{V_{C_{Vn}}}^2}{P_{C_{Vn}}}$$

6.3 - Impedance Angle of Variable Load

The variable load power factor can be arbitrated or known. For example, if it is a resistor the power factor is 1.

$$\theta_{C_V} = arc\ cos(FP_{C_V})$$

Where:

FP_{C_V} Variable load power factor

6.4 - Current of Variable Load

How:

$$\overrightarrow{I_{C_V}} = \frac{\frac{\overrightarrow{V_{Ts}}}{\sqrt{3}}}{\overrightarrow{Z_{C_V}}}$$

$$I_{C_V} = \frac{\left(\frac{V_{Ts}}{\sqrt{3}}, 0\right)}{\left(Z_{C_V}, \theta_{C_V}\right)}$$

$$\overrightarrow{I_{C_V}} = \left(\frac{V_{Ts}}{\sqrt{3}Z_{C_V}}, -\theta_{C_V}\right)$$

7 - MOTORS START

In loads composed of motor(s) starting, the impedance of the motor(s), at the start moment, is fixed. However, because it is a transient condition of the load, it will be treated differently.

7.1 - Starting Power of Motors

The impedance of motors on start is fixed. Therefore, its behavior is the same as loads with variable power.

$$P_{M_P} = \frac{{V_{Ts}}^2}{Z_{M_P}}$$

Where:

P_{M_P} Motor(s) power at starting (VA)

V_{Ts} Secondary voltage on transformer terminals (V)

Z_{M_P} Impedance of motor(s) at starting (Ω)

The rated power of the motor(s) at the starting is:

$$P_{M_{Pn}} = \frac{{V_{M_{Pn}}}^2}{Z_{M_{Pn}}}$$

Where:

$P_{M_{Pn}}$ Rated power of the motor(s) at starting (VA)

$V_{M_{Pn}}$ Rated voltage of the motor(s) (V)

$Z_{M_{Pn}}$ Rated impedance of the motor(s) at starting (Ω)

7.2 - Impedance of the Motor(s) at Starting

As the impedance of the motor(s) at the start is a fixed value:

$$Z_{M_P} = Z_{M_{Pn}}$$

$$Z_{M_P} = Z_{M_{Pn}} = \frac{{V_{M_{Pn}}}^2}{P_{M_{Pn}}}$$

As the rated starting power is not a data provided in the manufacturers' tables, we will use the rated starting current, i.e.:

$$P_{M_{Pn}} = \sqrt{3}V_{M_{Pn}}.I_{M_{Pn}}$$

Or:

$$Z_{M_P} = \frac{V_{M_{Pn}}}{\sqrt{3}I_{M_{Pn}}}$$

Where:

$P_{M_{Pn}}$ Rated power of the motor(s) at starting (VA)

$V_{M_{Pn}}$ Rated voltage of the motor(s) (V)

$I_{M_{Pn}}$ Motor(s) starting current at rated voltage (A)

7.3 - Impedance Angle of the Motor(s) at Starting

The value of power factor the motor(s) at starting can be estimated as defined according to the motor(s) data(s). So:

$$\theta_{M_P} = arc\ cos\left(FP_{M_P}\right)$$

Where:

FP_{M_P} Power factor of motors at starting

7.4 - Starting Current of the Motor(s)

$$\overrightarrow{I_{M_P}} = \frac{\frac{\overrightarrow{V_{Ts}}}{\sqrt{3}}}{\overrightarrow{Z_{M_P}}}$$

$$\overrightarrow{I_{M_P}} = \frac{(\frac{V_{Ts}}{\sqrt{3}}, 0)}{(Z_{M_P}, \theta_{M_P})}$$

$$\overrightarrow{I_{M_P}} = \left(\frac{V_{Ts}}{\sqrt{3}Z_{M_P}}, -\theta_{M_P}\right)$$

But how,

$$Z_{M_P} = \frac{V_{M_{Pn}}}{\sqrt{3}I_{M_{Pn}}}$$

$$\overrightarrow{I_{M_P}} = \left(\frac{I_{M_{Pn}}V_{Ts}}{V_{M_{Pn}}}, -\theta_{M_P}\right)$$

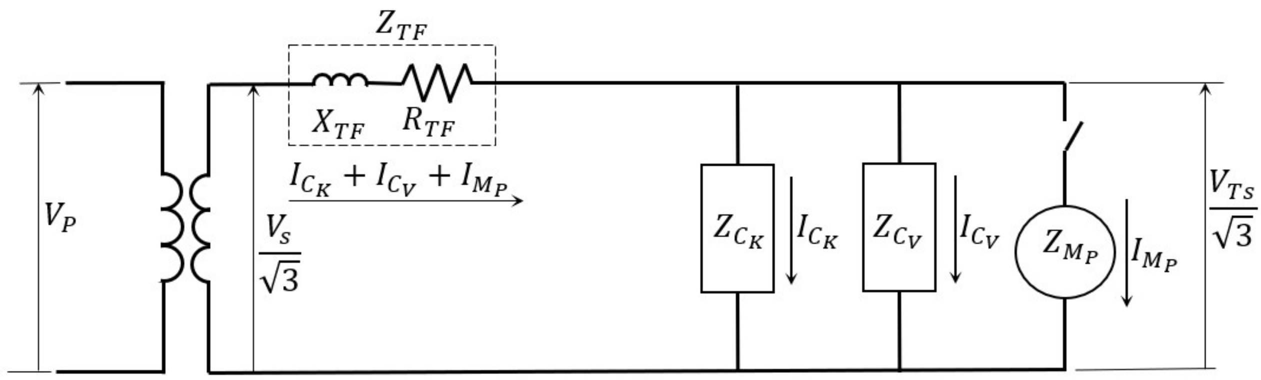
8 - OPERATING CONDITIONS THAT WILL BE ANALYZED

The operating conditions that will be analyze, during the permanent or transient feeding period of a load or set of loads, with or without starting of motors, shall take into account the voltage variations in the feed and the used tap of the primary winding.

The presentation of the calculations is done in detail to allow the understanding of the sequence and the concepts adopted, which can be used in the development of other applications. To monitor the development of the demonstrations, in addition to knowledge in electrotechnics, the user must also have knowledge of operations with complex numbers, in trigonometric and polar form.

9 - SECONDARY VOLTAGE OF THE TRANSFORMER

The following is the calculation of the voltage drop in the transformer secondary terminals when are know the power supply voltage, transformer and fed loads data.



The figure above represents the circuit of a transformer that feeds a load set composed of loads with constant power, loads with variable power and starting of motors.

From the circuit we can write the equation:

$$\frac{\overrightarrow{V_S}}{\sqrt{3}} = \frac{\overrightarrow{V_{T_S}}}{\sqrt{3}} + \overrightarrow{Z_{TS}}\left(\overrightarrow{I_{C_K}} + \overrightarrow{I_{C_V}} + \overrightarrow{I_{M_P}}\right)$$

Whereas:

$$\frac{\overrightarrow{V_{T_S}}}{\sqrt{3}} = \left(\frac{V_{T_S}}{\sqrt{3}} + j0\right)$$

Using the polar form to perform the calculations:

$$\frac{\overrightarrow{V_{Ts}}}{\sqrt{3}} = \left(\frac{V_{Ts}}{\sqrt{3}}, 0\right)$$

$$\frac{\overrightarrow{V_S}}{\sqrt{3}} = (\frac{V_{Ts}}{\sqrt{3}}, 0) + (Z_{TF}, \theta_{TF})\left(\frac{\left(\frac{V_{Ts}}{\sqrt{3}}, 0\right)}{(Z_{C_K}, \theta_{C_K})}\right) + \left(\frac{\left(\frac{V_{Ts}}{\sqrt{3}}, 0\right)}{(Z_{C_V}, \theta_{C_V})}\right) + \left(\frac{\left(\frac{V_{Ts}}{\sqrt{3}}, 0\right)}{(Z_{M_P}, \theta_{M_P})}\right)$$

$$\frac{\overrightarrow{V_S}}{\sqrt{3}} = \left(\frac{V_{Ts}}{\sqrt{3}}, 0\right) + (Z_{TF}, \theta_{TF})\left(\left(\frac{V_{Ts}}{\sqrt{3}Z_{C_K}}, -\theta_{C_K}\right) + \left(\frac{V_{Ts}}{\sqrt{3}Z_{C_V}}, -\theta_{C_V}\right) + \left(\frac{V_{Ts}}{\sqrt{3}Z_{M_P}}, -\theta_{M_P}\right)\right)$$

$$\frac{\overrightarrow{V_S}}{\sqrt{3}} = \left(\frac{V_{Ts}}{\sqrt{3}}, 0\right) + (Z_{TF}, \theta_{TF})\left(\frac{V_{Ts}}{\sqrt{3}Z_{C_K}}, -\theta_{C_K}\right) + (Z_{TF}, \theta_{TF})\left(\frac{V_{Ts}}{\sqrt{3}Z_{C_V}}, -\theta_{C_V}\right) +$$

$$+(Z_{TF}, \theta_{TF})\left(\frac{V_{Ts}}{\sqrt{3}Z_{M_P}}, -\theta_{M_P}\right)$$

$$\frac{\overrightarrow{V_S}}{\sqrt{3}} = \left(\frac{V_{Ts}}{\sqrt{3}}, 0\right) + \left(\frac{Z_{TF}V_{Ts}}{\sqrt{3}Z_{C_K}}, (\theta_{TF} - \theta_{C_K})\right) + \left(\frac{Z_{TF}V_{Ts}}{\sqrt{3}Z_{C_V}}, (\theta_{TF} - \theta_{C_V})\right) +$$
$$+ \left(\frac{Z_{TF}V_{Ts}}{\sqrt{3}Z_{M_P}}, (\theta_{TF} - \theta_{M_P})\right)$$

In the complexes form we have:

$$\frac{\overrightarrow{V_S}}{\sqrt{3}} = \frac{V_{Ts}}{\sqrt{3}} + j0 + \frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{C_K}} cos(\theta_{TF} - \theta_{C_K}) + j\frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{C_K}} sen(\theta_{TF} - \theta_{C_K}) + \frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{C_V}} cos(\theta_{TF} - \theta_{C_V}) +$$
$$+j\frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{C_V}} sen(\theta_{TF} - \theta_{C_V}) + \frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{M_P}} cos(\theta_{TF} - \theta_{M_P}) + j\frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{M_P}} sen(\theta_{TF} - \theta_{M_P})$$

$$\frac{\overrightarrow{V_S}}{\sqrt{3}} = \frac{V_{Ts}}{\sqrt{3}} + \frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{C_K}} cos(\theta_{TF} - \theta_{C_K}) + \frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{C_V}} cos(\theta_{TF} - \theta_{C_V}) + \frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{M_P}} cos(\theta_{TF} - \theta_{M_P}) +$$
$$+j\frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{C_K}} sen(\theta_{TF} - \theta_{C_K}) + j\frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{C_V}} sen(\theta_{TF} - \theta_{C_V}) + j\frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{M_P}} sen(\theta_{TF} - \theta_{M_P})$$

Replacing

$$Z_{C_K} = \frac{{V_{Ts}}^2}{P_{C_{Kn}}}$$

Have:

$$\frac{\overrightarrow{V_S}}{\sqrt{3}} = \frac{V_{Ts}}{\sqrt{3}} + \frac{V_{Ts}Z_{TF}}{\sqrt{3}\frac{{V_{Ts}}^2}{P_{C_{Kn}}}} cos(\theta_{TF} - \theta_{C_K}) + \frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{C_V}} cos(\theta_{TF} - \theta_{C_V}) + \frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{M_P}} cos(\theta_{TF} - \theta_{M_P}) +$$
$$+j\frac{V_{Ts}Z_{TF}}{\sqrt{3}\frac{{V_{Ts}}^2}{P_{C_{Kn}}}} sen(\theta_{TF} - \theta_{C_K}) + j\frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{C_V}} sen(\theta_{TF} - \theta_{C_V}) + j\frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{M_P}} sen(\theta_{TF} - \theta_{M_P})$$

$$\frac{\overrightarrow{V_S}}{\sqrt{3}} = \frac{V_{Ts}}{\sqrt{3}} + \frac{P_{C_{Kn}}Z_{TF}}{\sqrt{3}V_{Ts}} cos(\theta_{TF} - \theta_{C_K}) + \frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{C_V}} cos(\theta_{TF} - \theta_{C_V}) + \frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{M_P}} cos(\theta_{TF} - \theta_{M_P}) +$$
$$+j\frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{C_K}} sen(\theta_{TF} - \theta_{C_K}) + j\frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{C_V}} sen(\theta_{TF} - \theta_{C_V}) + j\frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{M_P}} sen(\theta_{TF} - \theta_{M_P})$$

$$\frac{\overrightarrow{V_S}}{\sqrt{3}} = \left[\frac{V_{Ts}}{\sqrt{3}} + \frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{C_V}} cos(\theta_{TF} - \theta_{C_V}) + \frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{M_P}} cos(\theta_{TF} - \theta_{M_P}) + \frac{P_{C_{Kn}}Z_{TF}}{\sqrt{3}V_{Ts}} cos(\theta_{TF} - \theta_{C_K})\right] +$$
$$+j\left[\frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{C_V}} sen(\theta_{TF} - \theta_{C_V}) + \frac{V_{Ts}Z_{TF}}{\sqrt{3}Z_{M_P}} sen(\theta_{TF} - \theta_{M_P}) + \frac{P_{C_{Kn}}Z_{TF}}{\sqrt{3}V_{Ts}} sen(\theta_{TF} - \theta_{C_K})\right]$$

$$+\frac{\overrightarrow{V_S}}{\sqrt{3}} = \left[\left(1 + \frac{Z_{TF}}{Z_{C_V}} cos(\theta_{TF} - \theta_{C_V}) + \frac{Z_{TF}}{Z_{M_P}} cos(\theta_{TF} - \theta_{M_P})\right)\frac{V_{Ts}}{\sqrt{3}} + \frac{P_{C_{Kn}}Z_{TF}}{\sqrt{3}V_{Ts}} cos(\theta_{TF} - \theta_{C_K})\right] +$$
$$+j\left[\left(\frac{Z_{TF}}{Z_{C_V}} sen(\theta_{TF} - \theta_{C_V}) + \frac{Z_{TF}}{Z_{M_P}} sen(\theta_{TF} - \theta_{M_P})\right)\frac{V_{Ts}}{\sqrt{3}} + \frac{P_{C_{Kn}}Z_{TF}}{\sqrt{3}V_{Ts}} sen(\theta_{TF} - \theta_{C_K})\right]$$

$$\left|\frac{\overrightarrow{V_S}}{\sqrt{3}}\right| = \left\|\left[\left(1+\frac{Z_{TF}}{Z_{C_V}}cos(\theta_{TF}-\theta_{C_V})+\frac{Z_{TF}}{Z_{M_P}}cos(\theta_{TF}-\theta_{M_P})\right)\frac{V_{Ts}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_{TF}}{\sqrt{3}V_{Ts}}cos(\theta_{TF}-\theta_{C_K})\right]+ +j\left[\left(\frac{Z_{TF}}{Z_{C_V}}sen(\theta_{TF}-\theta_{C_V})+\frac{Z_{TF}}{Z_{M_P}}sen(\theta_{TF}-\theta_{M_P})\right)\frac{V_{Ts}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_{TF}}{\sqrt{3}V_{Ts}}sen(\theta_{TF}-\theta_{C_K})\right]\right\|$$

$$\left(\frac{V_S}{\sqrt{3}}\right)^2 = \left[\left(1+\frac{Z_{TF}}{Z_{C_V}}cos(\theta_{TF}-\theta_{C_V})+\frac{Z_{TF}}{Z_{M_P}}cos(\theta_{TF}-\theta_{M_P})\right)\frac{V_{Ts}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_{TF}}{\sqrt{3}V_{Ts}}cos(\theta_{TF}-\theta_{C_K})\right]^2 + +j\left[\left(\frac{Z_{TF}}{Z_{C_V}}sen(\theta_{TF}-\theta_{C_V})++\frac{Z_{TF}}{Z_{M_P}}sen(\theta_{TF}-\theta_{M_P})\right)\frac{V_{Ts}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_{TF}}{\sqrt{3}V_{Ts}}sen(\theta_{TF}-\theta_{C_K})\right]^2$$

Or:

$$\left[\left(1+\frac{Z_{TF}}{Z_{C_V}}cos(\theta_{TF}-\theta_{C_V})+\frac{Z_{TF}}{Z_{M_P}}cos(\theta_{TF}-\theta_{M_P})\right)\frac{V_{Ts}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_{TF}}{\sqrt{3}V_{Ts}}cos(\theta_{TF}-\theta_{C_K})\right]^2 + +j\left[\left(\frac{Z_{TF}}{Z_{C_V}}sen(\theta_{TF}-\theta_{C_V})++\frac{Z_{TF}}{Z_{M_P}}sen(\theta_{TF}-\theta_{M_P})\right)\frac{V_{Ts}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_{TF}}{\sqrt{3}V_{Ts}}sen(\theta_{TF}-\theta_{C_K})\right]^2 - \frac{{V_S}^2}{3} = 0$$

$$\left[\left(1+\left(\frac{cos(\theta_{TF}-\theta_{C_V})}{Z_{C_V}}+\frac{cos(\theta_{TF}-\theta_{M_P})}{Z_{M_P}}\right)Z_{TF}\right)\frac{V_{Ts}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_{TF}}{\sqrt{3}V_{Ts}}cos(\theta_{TF}-\theta_{C_K})\right]^2 + +\left[\left(\frac{sen(\theta_{TF}-\theta_{C_V})}{Z_{C_V}}+\frac{sen(\theta_{TF}-\theta_{M_P})}{Z_{M_P}}\right)Z_{TF}\frac{V_{Ts}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_{TF}}{\sqrt{3}V_{Ts}}sen(\theta_{TF}-\theta_{C_K})\right]^2 - \frac{{V_S}^2}{3} = 0$$

$$\left[\left(1+\left(\frac{cos(\theta_{TF}-\theta_{C_V})}{Z_{C_V}}+\frac{cos(\theta_{TF}-\theta_{M_P})}{Z_{M_P}}\right)Z_{TF}\right)\frac{V_{Ts}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_{TF}}{3\frac{V_{Ts}}{\sqrt{3}}}cos(\theta_{TF}-\theta_{C_K})\right]^2 + +\left[\left(\frac{sen(\theta_{TF}-\theta_{C_V})}{Z_{C_V}}+\frac{sen(\theta_{TF}-\theta_{M_P})}{Z_{M_P}}\right)Z_{TF}\frac{V_{Ts}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_{TF}}{3\frac{V_{Ts}}{\sqrt{3}}}sen(\theta_{TF}-\theta_{C_K})\right]^2 - \frac{{V_S}^2}{3} = 0$$

If:

$$\boldsymbol{x = \frac{V_{Ts}}{\sqrt{3}}}$$

$$\boldsymbol{a_1 = 1+\left(\frac{cos(\theta_{TF}-\theta_{C_V})}{Z_{C_V}}+\frac{cos(\theta_{TF}-\theta_{M_P})}{Z_{M_P}}\right)Z_{TF}}$$

$$\boldsymbol{b_1 = \frac{P_{C_{Kn}}Z_{TF}}{3}cos(\theta_{TF}-\theta_{C_K})}$$

$$\boldsymbol{c_1 = \left(\frac{sen(\theta_{TF}-\theta_{C_V})}{Z_{C_V}}+\frac{sen(\theta_{TF}-\theta_{M_P})}{Z_{M_P}}\right)Z_{TF}}$$

$$\boldsymbol{d_1 = \frac{P_{C_{Kn}}Z_{TF}}{3}sen(\theta_{TF}-\theta_{C_K})}$$

$$\boldsymbol{e_1 = -\frac{V_S{}^2}{3}}$$

We can write the featured equation as:

$$\left[a_1x + \frac{b_1}{x}\right]^2 - \left[c_1x + \frac{d_1}{x}\right]^2 + e_1 = 0$$

$$\left(a_1{}^2x^2 + 2a_1x\frac{b_1}{x} + \left(\frac{b_1}{x}\right)^2\right) + \left(c_1{}^2x^2 + 2c_1x\frac{d_1}{x} + \left(\frac{d_1}{x}\right)^2\right) + e_1 = 0$$

$$a_1{}^2x^2 + 2a_1x\frac{b_1}{x} + \frac{b_1{}^2}{x^2} + c_1{}^2x^2 + 2c_1x\frac{d_1}{x} + \frac{d_1{}^2}{x^2} + e_1 = 0$$

$$a_1{}^2x^2 + c_1{}^2x^2 + 2a_1x\frac{b_1}{x} + 2c_1x\frac{d_1}{x} + \frac{b_1{}^2}{x^2} + \frac{d_1{}^2}{x^2} + e_1 = 0$$

$$(a_1{}^2 + c_1{}^2)x^2 + 2a_1b_1 + 2c_1d_1 + \frac{b_1{}^2 + d_1{}^2}{x^2} + e_1 = 0$$

$$(a_1{}^2 + c_1{}^2)x^2 + 2(a_1b_1 + c_1d_1) + e_1 + \frac{b_1{}^2 + d_1{}^2}{x^2} = 0$$

Multiplying by x^2

$$(a_1{}^2 + c_1{}^2)x^4 + (2(a_1b_1 + c_1d_1) + e_1)x^2 + b_1{}^2 + d_1{}^2 = 0$$

The solution of the above equation will be the real and positive root of the equation below:

$$\boldsymbol{ax^4 + cx^2 + e = 0}$$

Where:

$$\boldsymbol{a = a_1{}^2 + c_1{}^2}$$

$$\boldsymbol{c = 2(a_1b_1 + c_1d_1) + e_1}$$

$$\boldsymbol{e = b_1{}^2 + d_1{}^2}$$

$$\frac{V_{Ts}}{\sqrt{3}} = \sqrt{\frac{-c + \sqrt{c^2 - 4ae}}{2a}}$$

$$\boxed{\boldsymbol{V_{Ts} = \sqrt{3}\sqrt{\frac{-c + \sqrt{c^2 - 4ae}}{2a}}}}$$

This is the formula that calculates the voltage in the secondary terminals of transformer.

10 - PREPARATION OF EXCEL SPREADSHEETS

Excel spreadsheets were prepared based on the concepts developed in this technical information. The theoretical part is not indispensable for its use, but it is important for understanding the problem and possible development of further studies.

Two spreadsheets were elaborated to determine the voltage in the secondary terminals of transformer, one complete and the other simplified. In the complete worksheet are indicated the formulas of all terms used in the calculation and, in the simplified, which is identical to the complete, only the basic information's are visible.

Because the two spreadsheets are identical, the fields and information in the completed spreadsheet are not visible in the simplified spreadsheet, but they are hidden, and still active. Therefore, any changes, or introduction of information, must be done carefully so as not to corrupt the file. The two spreadsheets are in a single file.

As in an installation there may be loads with nominal voltages different from those of the system, there are fields to provide this information. For example, there may be motors and loads with rated voltages of 220V, 380V, 440V, 460V, 480, etc.

The spreadsheets are for all cases, that is, the transformer may or may not be with initial load, this initial load may be formed by constant or variable load. The load to be applied, being the transformer with initial load, can be one or more types of loads, constants, variables, or motor (s).

The reference made to the motors(s) is intended to allow a single motor or set of motors to be considered, the data of which must be defined. For example, if you want to know exactly the behavior of the system with the simultaneous start of two motors with different characteristics. In this case, the starting current and power factor equivalent to the two motors must be determined to insert them into the sheet.