FEEDERS

Voltage Calculation on the Load and Source

INDEX **PAG.**

1 - PURPOSE

The purpose of this document is to study the feeder circuits to calculate the voltage in the load when the voltage in the source is known and, to calculate the voltage of the source to meet the voltage defined in the load. Based on this study will be prepared Excel spreadsheets to perform the calculations.

2 - REFERENCE DOCUMENTS

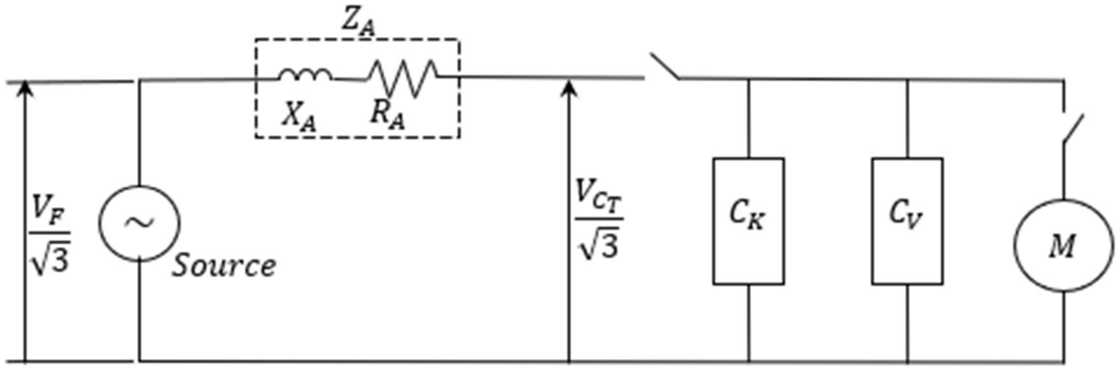
2.1 - Spreadsheets

PL.EL.SA.CA.04.R1 Feeders - Load Voltage Calculation

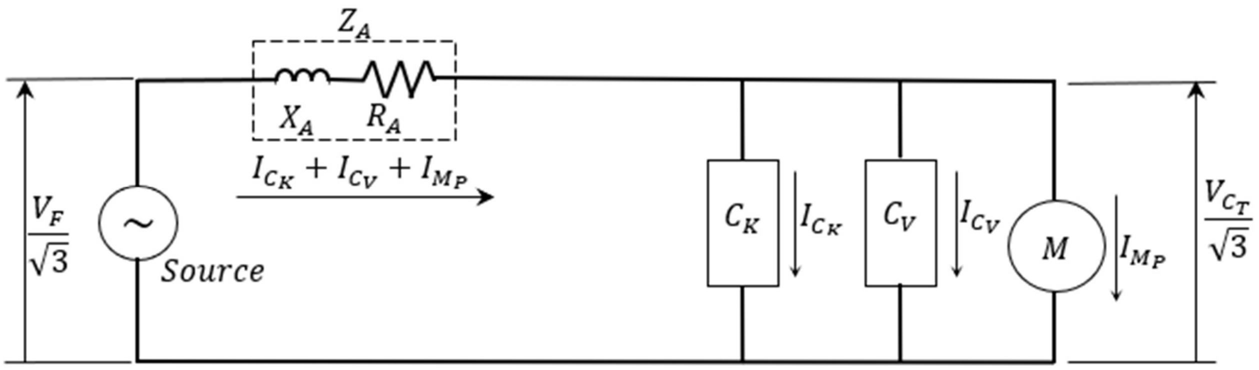
PL.EL.SA.CA.05.R1 Feeders - Source Voltage Calculation

3 - BASIC CIRCUIT

The figure above represents the circuit, whose source is a star circuit, solidly grounded, to feed the various types of charges that are usually found in practice. The loads can be of constant power, variable power and starting of motors.



Before the load is switched on, the voltage at the terminal of the feeder circuit V_{C_T} will be equal to the voltage of the source V_F.



The circuit figure above represents the circuit at the instant the loads are turned on.

Being:

V_F Source voltage (V)

V_{C_T} Load voltage (V)

I_{C_K} Current of constant power load C_K (A)

I_{C_V} Current of variable power load C_V (A)

I_{M_P} Starting current of the motor(s) (A)

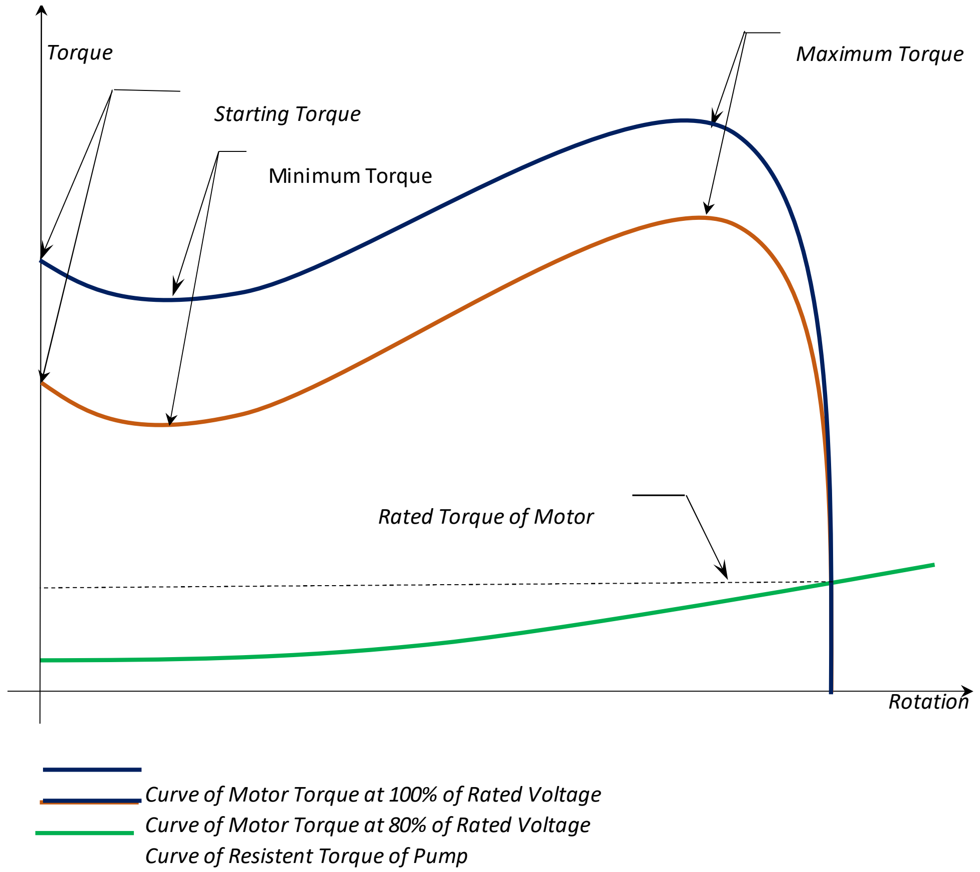
Z_A Feeder impedance (Ω)

R_A Feeder resistance (Ω)

X_A Feeder reactance (Ω)

4 - LOADS OF CONSTANT POWER

At constant power loads, the current varies as a function of the voltage to keep the power constant. In this case are, for example, battery chargers, communication systems and, mainly, induction motors. Induction motors have the characteristic of keeping the rotation practically constant with the voltage variation (see figure below).



$Pwer = Force\ x\ Speed$ or $Power = Torque\ x\ Angular\ Velocity$

Note that the motor conjugate varies during motor start-up and is different for each voltage value, but the conjugate and rotation remain constant during operation. Therefore, as the load (Force or Torque) and the Speed remain constant, the Power also remains constant, that is, the variation of the current is inversely proportional to the variation of the voltage.

4.1 - Constant Load Power

The power of the constant load in the circuit is given by:

$$P_{C_K} = \frac{{V_{Ts}}^2}{Z_{C_K}}$$

Where:

P_{C_K}- Constant load power (VA)

V_{Ts}- Voltage at transformer secondary terminals (V)

Z_{C_K}– Impedance of constant load (Ω)

At constant power load, the power is equal to the rated power of the load, i.e.:

$$P_{C_{Kn}} = \frac{{V_{C_{Kn}}}^2}{Z_{C_{Kn}}}$$

$P_{C_{Kn}}$- Rated power of constant load (VA)

$V_{C_{Kn}}$- Rated voltage of constant load (V)

$Z_{C_{Kn}}$- Rated Impedance of constant load (Ω)

4.2 - Impedance of Constant Load

Since the power of the load is constant:

$$Z_{C_K} = \frac{{V_{Ts}}^2}{P_{C_{Kn}}}$$

4.3 - Impedance Angle of the Constant Load

Since the load power factor is usually an arbitrated value, for example, equal to 0.85,

$$\theta_{C_K} = arc\ cos\left(FP_{C_K}\right)$$

Where:

FP_{C_K} Constant load power factor

4.4 - Constant Load Current

How:

$$\overrightarrow{I_{C_K}} = \frac{\frac{\overrightarrow{V_{Ts}}}{\sqrt{3}}}{\overrightarrow{Z_{C_K}}}$$

$$\overrightarrow{I_{C_K}} = \frac{(\frac{V_{Ts}}{\sqrt{3}}, 0)}{(Z_{C_K}, \theta_{C_K})}$$

$$\overrightarrow{I_{C_K}} = \left(\frac{V_{Ts}}{\sqrt{3}Z_{C_K}}, -\theta_{C_K}\right)$$

But as,

$$Z_{C_K} = \frac{{V_{Ts}}^2}{P_{C_{Kn}}}$$

$$\overrightarrow{I_{C_K}} = \left(\frac{P_{C_{Kn}}}{\sqrt{3}V_{Ts}}, -\theta_{C_K}\right)$$

5 - LOADS OF VARIABLE POWER

In variable power loads the impedance is a constant value. Therefore, the variation of the voltage causes the variation of the current as a function of the impedance of the load. In this case we can consider, for example, loads composed of transformers, reactors, and resistors. At these loads the current is directly proportional to the voltage variation.

5.1 - Variable Load Power

In the variable power load, we have:

$$P_{C_V} = \frac{{V_{Ts}}^2}{Z_{C_V}}$$

Where:

P_{C_V} Variable load power (VA)

V_{Ts} Voltage at transformer secondary terminals (V)

Z_{C_V} Impedance of variable load (Ω)

The rated power of the variable load is:

$$P_{C_{Vn}} = \frac{{V_{C_{Vn}}}^2}{Z_{C_{Vn}}}$$

Where:

$P_{C_{Vn}}$ Rated power of variable load (VA)

$V_{C_{Vn}}$ Rated voltage of variable load (V)

$Z_{C_{Vn}}$ Rated impedance of variable load (Ω)

5.2 - Impedance of Variable Load

The load is variable because the impedance is constant (e.g., resistor), so:

$$Z_{C_V} = Z_{C_{Vn}}$$

$$Z_{C_V} = Z_{C_{Vn}} = \frac{{V_{C_{Vn}}}^2}{P_{C_{Vn}}}$$

$$Z_{C_V} = \frac{{V_{C_{Vn}}}^2}{P_{C_{Vn}}}$$

5.3 - Impedance Angle of Variable Load

The power factor of the variable load can be arbitrated or known. For example, if it is a resistor, the power factor is 1.

$$\theta_{C_V} = arc\ cos\left(FP_{C_V}\right)$$

Where:

FP_{C_V} Power factor of variable load

5.4 - Current of Variable Load

How:

$$\overrightarrow{I_{C_V}} = \frac{\frac{\overrightarrow{V_{Ts}}}{\sqrt{3}}}{\overrightarrow{Z_{C_V}}}$$

$$I_{C_V} = \frac{\left(\frac{V_{Ts}}{\sqrt{3}}, 0\right)}{\left(Z_{C_V}, \theta_{C_V}\right)}$$

$$\overrightarrow{I_{C_V}} = \left(\frac{V_{Ts}}{\sqrt{3}Z_{C_V}}, -\theta_{C_V}\right)$$

6 - STARTING OF MOTORS

In loads composed starting of motor(s), the impedance of the motor(s) at the moment of starting is fix. However, because it is a transitory condition of the load, it will be treated differently.

6.1 - Power of Motors Starter

As in the starting of motor(s) the impedance of the motors is fix, their behavior is the same as the loads with variable power.

$$P_{M_P} = \frac{{V_{Ts}}^2}{Z_{M_P}}$$

Where:

P_{M_P} Power of motor(s) on starting (VA)

V_{Ts} Voltage at the transformer secondary terminals (V)

Z_{M_P} Impedance of the motor(s) at starting (Ω)

The rated power of the motor(s) at start is:

$$P_{M_{Pn}} = \frac{{V_{M_{Pn}}}^2}{Z_{M_{Pn}}}$$

Where:

$P_{M_{Pn}}$ Rated power of the motor(s) at start (VA)

$V_{M_{Pn}}$ Rated voltage of the motor(s) (V)

$Z_{M_{Pn}}$ Rated impedance of the motor(s) at start (Ω)

6.2 - Impedance of the Motor(s) at the Start

As the impedance of the motor(s) at start is a fixed value:

$$Z_{M_P} = Z_{M_{Pn}}$$

$$Z_{M_P} = Z_{M_{Pn}} = \frac{{V_{M_{Pn}}}^2}{P_{M_{Pn}}}$$

As the rated starting power is not a given data in the manufacturers' tables, we will use the nominal starting current, that is:

$$P_{M_{Pn}} = \sqrt{3}V_{M_{Pn}}.I_{M_{Pn}}$$

Or:

$$Z_{M_P} = \frac{V_{M_{Pn}}}{\sqrt{3}I_{M_{Pn}}}$$

Where:

$P_{M_{Pn}}$ Rated power of the motor(s) at start (VA)

$V_{M_{Pn}}$ Rated voltage of motor(s) (V)

$I_{M_{Pn}}$ Starting current of the motor(s) at rated voltage (A)

6.3 - Impedance Angle of the Motor(s) at the Start

The power factor value(s) of the motor(s) at start can be estimated as defined as a function of the motor(s) data. So:

$$\theta_{M_P} = arc\ cos\left(FP_{M_P}\right)$$

Where:

FP_{M_P} Power factor of the motor(s) at start

6.4 - Starting Current of Motor(s)

$$\overrightarrow{I_{M_P}} = \frac{\frac{\overrightarrow{V_{Ts}}}{\sqrt{3}}}{\overrightarrow{Z_{M_P}}}$$

$$\overrightarrow{I_{M_P}} = \frac{(\frac{V_{Ts}}{\sqrt{3}}, 0)}{(Z_{M_P}, \theta_{M_P})}$$

$$\overrightarrow{I_{M_P}} = \left(\frac{V_{Ts}}{\sqrt{3}Z_{M_P}}, -\theta_{M_P}\right)$$

But as,

$$Z_{M_P} = \frac{V_{M_{Pn}}}{\sqrt{3}I_{M_{Pn}}}$$

$$\overrightarrow{I_{M_P}} = \left(\frac{I_{M_{Pn}}V_{Ts}}{V_{M_{Pn}}}, -\theta_{M_P}\right)$$

7 - FEEDERS

Feeders are usually cables or buses connected to equipment such as transformers, switchboards, diesel generator sets, etc.

7.1 - Feeders Impedance

The impedance of the feeders is obtained according to the data of the resistances and reactances defined by the user, based on the conditions of use and information of the manufacturers and recommendations of the standards.

$$Z_A = \sqrt{{R_A}^2 + {X_A}^2}$$

Where:

Z_A Feeder impedance (Ω)

R_A Feeder resistance (Ω)

X_A Feeder reactance (Ω)

7.2 - Resistance and Reactance of the Feeders

The values of impedance, resistance and reactance are defined as a function of the type of cable used, circuit length, installation conditions, ambient temperature, quantity of cables per phase, design criteria and other conditions defined by the user.

$$R_A = R_a \frac{l}{n}$$

Where:

R_a Resistance of cable(s) under installation conditions (Ω/km)

l Length of cable(s) (m)

n Number of cables per phase

Reactance of the cable(s)

$$X_A = X_a \frac{l}{n}$$

X_a Reactance of the cable(s) under installation conditions (Ω/km)

l Length of cable(s) (m)

n Number of cables per phase

7.3 - Impedance Angle

$$\theta_A = arc\ tan \frac{X_a}{R_a}$$

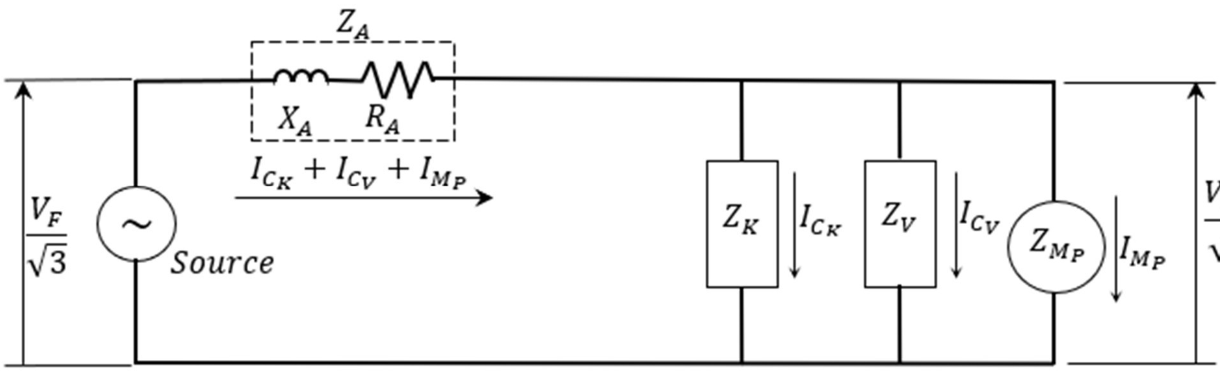
8 - OPERATING CONDITIONS THAT WILL BE ANALYZED

The operating conditions that will be analyzed, during the permanent or transitory period of the feeding of a load or set of loads, with or without starting of motor(s), will consider the data of the loads and the feeders.

The presentation of the calculations is done in detail to allow the perfect understanding of the sequence and concepts adopted, which can be used in the development of other applications. To follow the development user should have only knowledge of complex numbers, in trigonometric and polar form and, of course, electrotechnical.

9 - CALCULATION OF THE VOLTAGE IN THE LOAD

The following calculation is the calculation of the voltage at the terminals of the load when the voltage of the source is known.



The figure above represents the circuit of a feeder that feeds a load composed of load with constant power, load with variable power and starting of motor(s).

$$\frac{\overrightarrow{\boldsymbol{V_F}}}{\sqrt{\boldsymbol{3}}} = \frac{\overrightarrow{\boldsymbol{V_{C_T}}}}{\sqrt{\boldsymbol{3}}} + \overrightarrow{\boldsymbol{Z_A}}\left(\overrightarrow{\boldsymbol{I_{C_K}}} + \overrightarrow{\boldsymbol{I_{C_V}}} + \overrightarrow{\boldsymbol{I_{M_P}}}\right)$$

Whereas:

$$\frac{\overrightarrow{\boldsymbol{V_{C_T}}}}{\sqrt{\boldsymbol{3}}} = \left(\frac{\boldsymbol{V_{C_T}}}{\sqrt{\boldsymbol{3}}} + \boldsymbol{j0}\right)$$

Using the polar shape to perform the calculations:

$$\frac{\overrightarrow{V_{C_T}}}{\sqrt{3}} = \left(\frac{V_{C_T}}{\sqrt{3}}, 0\right)$$

$$\frac{\overrightarrow{V_F}}{\sqrt{3}} = \left(\frac{V_{C_T}}{\sqrt{3}}, 0\right) + (Z_A, \theta_A)\left(\frac{\left(\frac{V_{C_T}}{\sqrt{3}}, 0\right)}{(Z_{C_K}, \theta_{C_K})}\right) + \left(\frac{\left(\frac{V_{C_T}}{\sqrt{3}}, 0\right)}{(Z_{C_V}, \theta_{C_V})}\right) + \left(\frac{\left(\frac{V_{C_T}}{\sqrt{3}}, 0\right)}{(Z_{M_P}, \theta_{M_P})}\right)$$

$$\frac{\overrightarrow{V_F}}{\sqrt{3}} = \left(\frac{V_{C_T}}{\sqrt{3}}, 0\right) + (Z_A, \theta_A)\left(\left(\frac{V_{C_T}}{\sqrt{3}Z_{C_K}}, -\theta_{C_K}\right) + \left(\frac{V_{C_T}}{\sqrt{3}Z_{C_V}}, -\theta_{C_V}\right) + \left(\frac{V_{C_T}}{\sqrt{3}Z_{M_P}}, -\theta_{M_P}\right)\right)$$

$$\frac{\overrightarrow{V_F}}{\sqrt{3}} = \left(\frac{V_{C_T}}{\sqrt{3}}, 0\right) + (Z_A, \theta_A)\left(\frac{V_{C_T}}{\sqrt{3}Z_{C_K}}, -\theta_{C_K}\right) + (Z_A, \theta_A)\left(\frac{V_{C_T}}{\sqrt{3}Z_{C_V}}, -\theta_{C_V}\right) + (Z_A, \theta_A)\left(\frac{V_{C_T}}{\sqrt{3}Z_{M_P}}, -\theta_{M_P}\right)$$

$$\frac{\overrightarrow{V_F}}{\sqrt{3}} = \left(\frac{V_{C_T}}{\sqrt{3}}, 0\right) + \left(\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_K}}, (\theta_A - \theta_{C_K})\right) + \left(\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}, (\theta_A - \theta_{C_V})\right) + + \left(\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}, (\theta_A - \theta_{M_P})\right)$$

In the form of complexes, we have:

$$\frac{\overrightarrow{V_F}}{\sqrt{3}} = \frac{V_{C_T}}{\sqrt{3}} + j0 + \frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_K}} cos(\theta_A - \theta_{C_K}) + j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_K}} sen(\theta_A - \theta_{C_K}) + \frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}} cos(\theta_A - \theta_{C_V}) +$$

$$+j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}} sen(\theta_A - \theta_{C_V}) + \frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}} cos(\theta_A - \theta_{M_P}) + j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}} sen(\theta_A - \theta_{M_P})$$

$$\frac{\overrightarrow{V_F}}{\sqrt{3}} = \frac{V_{C_T}}{\sqrt{3}} + \frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_K}} cos(\theta_A - \theta_{C_K}) + \frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}} cos(\theta_A - \theta_{C_V}) + + \frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}} cos(\theta_A - \theta_{M_P}) +$$

$$+j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_K}} sen(\theta_A - \theta_{C_K}) + +j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}} sen(\theta_A - \theta_{C_V}) + j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}} sen(\theta_A - \theta_{M_P})$$

Replacing

$$Z_{C_K} = \frac{{V_{C_T}}^2}{P_{C_{Kn}}}$$

$$\frac{\overrightarrow{V_F}}{\sqrt{3}} = \frac{V_{C_T}}{\sqrt{3}} + \frac{Z_A V_{C_T}}{\sqrt{3}\frac{{V_{C_T}}^2}{P_{C_{Kn}}}} cos(\theta_A - \theta_{C_K}) + \frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}} cos(\theta_A - \theta_{C_V}) + \frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}} cos(\theta_A - \theta_{M_P}) +$$

$$+j\frac{Z_A V_{C_T}}{\sqrt{3}\frac{{V_{C_T}}^2}{P_{C_{Kn}}}} sen\left(\theta_A-\theta_{C_K}\right)+j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}} sen\left(\theta_A-\theta_{C_V}\right)+j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}} sen\left(\theta_A-\theta_{M_P}\right)$$

$$\frac{\overrightarrow{V_F}}{\sqrt{3}}=\left[\frac{V_{C_T}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}cos\left(\theta_A-\theta_{C_K}\right)+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}cos\left(\theta_A-\theta_{C_V}\right)+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}cos\left(\theta_A-\theta_{M_P}\right)\right]+$$

$$+j\left[\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}sen\left(\theta_A-\theta_{C_K}\right)+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}sen\left(\theta_A-\theta_{C_V}\right)+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}sen\left(\theta_A-\theta_{M_P}\right)\right]$$

$$\left|\frac{\overrightarrow{V_F}}{\sqrt{3}}\right|=\left|\frac{V_{C_T}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}cos\left(\theta_A-\theta_{C_K}\right)+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}cos\left(\theta_A-\theta_{C_V}\right)+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}cos\left(\theta_A-\theta_{M_P}\right)\right]+$$

$$+j\left[\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}sen\left(\theta_A-\theta_{C_K}\right)++\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}sen\left(\theta_A-\theta_{C_V}\right)+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}sen\left(\theta_A-\theta_{M_P}\right)\right]\Bigg|$$

$$\left(\frac{V_F}{\sqrt{3}}\right)^2=\left[\left(1+\frac{Z_A}{Z_{C_V}}cos\left(\theta_A-\theta_{C_V}\right)+\frac{Z_A}{Z_{M_P}}cos\left(\theta_A-\theta_{M_P}\right)\right)\frac{V_{C_T}}{\sqrt{3}}++\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}cos\left(\theta_A-\theta_{C_K}\right)\right]^2+$$

$$+j\left[\left(\frac{Z_A}{Z_{C_V}}sen\left(\theta_A-\theta_{C_V}\right)++\frac{Z_A}{Z_{M_P}}sen\left(\theta_A-\theta_{M_P}\right)\right)\frac{V_{C_T}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}sen\left(\theta_A-\theta_{C_K}\right)\right]^2$$

Finally:

$$\left[\left(1+\frac{Z_A}{Z_{C_V}}cos\left(\theta_A-\theta_{C_V}\right)+\frac{Z_A}{Z_{M_P}}cos\left(\theta_A-\theta_{M_P}\right)\right)\frac{V_{C_T}}{\sqrt{3}}++\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}cos\left(\theta_A-\theta_{C_K}\right)\right]^2+$$

$$+j\left[\left(\frac{Z_A}{Z_{C_V}}sen\left(\theta_A-\theta_{C_V}\right)++\frac{Z_A}{Z_{M_P}}sen\left(\theta_A-\theta_{M_P}\right)\right)\frac{V_{C_T}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}sen\left(\theta_A-\theta_{C_K}\right)\right]^2-\frac{{V_F}^2}{3}=0$$

$$\left[\left(1+\left(\frac{cos\left(\theta_A-\theta_{C_V}\right)}{Z_{C_V}}+\frac{cos\left(\theta_A-\theta_{M_P}\right)}{Z_{M_P}}\right)Z_A\right)\frac{V_{C_T}}{\sqrt{3}}++\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}cos\left(\theta_A-\theta_{C_K}\right)\right]^2+$$

$$+\left[\left(\frac{sen\left(\theta_A-\theta_{C_V}\right)}{Z_{C_V}}++\frac{sen\left(\theta_A-\theta_{M_P}\right)}{Z_{M_P}}\right)Z_A\frac{V_C}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}sen\left(\theta_A-\theta_{C_K}\right)\right]^2-\frac{{V_F}^2}{3}=0$$

$$\left[\left(1+\left(\frac{cos\left(\theta_A-\theta_{C_V}\right)}{Z_{C_V}}+\frac{cos\left(\theta_A-\theta_{M_P}\right)}{Z_{M_P}}\right)Z_A\right)\frac{V_{C_T}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_A}{3\frac{V_{C_T}}{\sqrt{3}}}cos\left(\theta_A-\theta_{C_K}\right)\right]^2+$$

$$+\left[\left(\frac{sen\left(\theta_A-\theta_{C_V}\right)}{Z_{C_V}}+\frac{sen\left(\theta_A-\theta_{M_P}\right)}{Z_{M_P}}\right)Z_A\frac{V_{C_T}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_A}{3\frac{V_{C_T}}{\sqrt{3}}}sen\left(\theta_A-\theta_{C_K}\right)\right]^2-\frac{{V_F}^2}{3}=0$$

If:

$$x=\frac{V_{C_T}}{\sqrt{3}}$$

$$a_1 = 1 + \left(\frac{cos(\theta_A - \theta_{C_V})}{Z_{C_V}} + \frac{cos(\theta_A - \theta_{M_P})}{Z_{M_P}}\right) Z_A$$

$$b_1 = \frac{P_{C_{Kn}} Z_A}{3} cos(\theta_A - \theta_{C_K})$$

$$c_1 = \left(\frac{sen(\theta_A - \theta_{C_V})}{Z_{C_V}} + \frac{sen(\theta_A - \theta_{M_P})}{Z_{M_P}}\right) Z_A$$

$$d_1 = \frac{P_{C_{Kn}} Z_A}{3} sen(\theta_A - \theta_{C_K})$$

$$e_1 = -\frac{{V_F}^2}{3}$$

We can write the highlighted equation as:

$$\left[a_1 x + \frac{b_1}{x}\right]^2 - \left[c_1 x + \frac{d_1}{x}\right]^2 + e_1 = 0$$

$$\left({a_1}^2 x^2 + 2a_1 x\frac{b_1}{x} + \left(\frac{b_1}{x}\right)^2\right) + \left({c_1}^2 x^2 + 2c_1 x\frac{d_1}{x} + \left(\frac{d_1}{x}\right)^2\right) + e_1 = 0$$

$${a_1}^2 x^2 + 2a_1 x\frac{b_1}{x} + \frac{{b_1}^2}{x^2} + {c_1}^2 x^2 + 2c_1 x\frac{d_1}{x} + \frac{{d_1}^2}{x^2} + e_1 = 0$$

$${a_1}^2 x^2 + {c_1}^2 x^2 + 2a_1 x\frac{b_1}{x} + 2c_1 x\frac{d_1}{x} + \frac{{b_1}^2}{x^2} + \frac{{d_1}^2}{x^2} + e_1 = 0$$

$$({a_1}^2 + {c_1}^2)x^2 + 2a_1 b_1 + 2c_1 d_1 + \frac{{b_1}^2 + {d_1}^2}{x^2} + e_1 = 0$$

$$({a_1}^2 + {c_1}^2)x^2 + 2(a_1 b_1 + c_1 d_1) + e_1 + \frac{{b_1}^2 + {d_1}^2}{x^2} = 0$$

Multiplying by x^2

$$\mathbf{({a_1}^2 + {c_1}^2)x^4 + (2(a_1 b_1 + c_1 d_1) + e_1)x^2 + {b_1}^2 + {d_1}^2 = 0}$$

The solution of the above equation will be the real, positive root of the equation below:

$$\boldsymbol{ax^4 + cx^2 + e = 0}$$

Where:

$$\boldsymbol{a = {a_1}^2 + {c_1}^2}$$

$$\boldsymbol{c = 2(a_1 b_1 + c_1 d_1) + e_1}$$

$$\boldsymbol{e = {b_1}^2 + {d_1}^2}$$

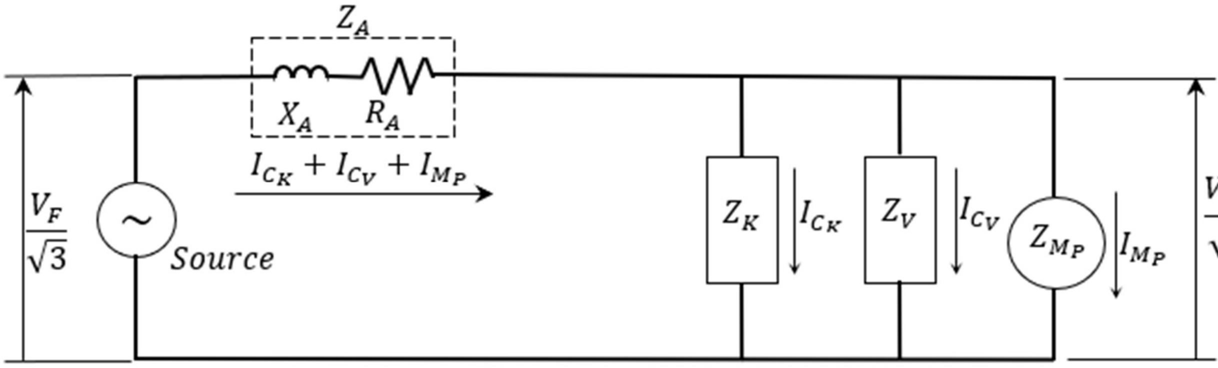
$$\frac{V_C}{\sqrt{3}} = \sqrt{\frac{-c + \sqrt{c^2 - 4ae}}{2a}}$$

$$\boldsymbol{V_{C_T} = \sqrt{3}\sqrt{\frac{-c + \sqrt{c^2 - 4ae}}{2a}}}$$

The above formula provides the value of the voltage at the load terminals considering the data from the source, feeders, and loads.

10 - CALCULATION OF THE SOURCE VOLTAGE

The following calculation is the calculation of the voltage required at the source to meet a defined voltage on the load.



The figure above represents the circuit of a feeder that feeds a load composed of load with constant power, load with variable power and starting of motor(s).

$$\frac{\overrightarrow{V_F}}{\sqrt{3}} = \frac{\overrightarrow{V_{C_T}}}{\sqrt{3}} + \overrightarrow{Z_A}\left(\overrightarrow{I_{C_K}} + \overrightarrow{I_{C_V}} + \overrightarrow{I_{M_P}}\right)$$

Whereas:

$$\frac{\overrightarrow{V_{C_T}}}{\sqrt{3}} = \left(\frac{V_{C_T}}{\sqrt{3}} + j0\right)$$

Using the polar shape to perform the calculations:

$$\frac{\overrightarrow{V_{C_T}}}{\sqrt{3}} = \left(\frac{V_{C_T}}{\sqrt{3}}, 0\right)$$

$$\frac{\overrightarrow{V_F}}{\sqrt{3}} = \left(\frac{V_{C_T}}{\sqrt{3}}, 0\right) + (Z_A, \theta_A)\left(\frac{\left(\frac{V_{C_T}}{\sqrt{3}}, 0\right)}{(Z_{C_K}, \theta_{C_K})}\right) + \left(\frac{\left(\frac{V_{C_T}}{\sqrt{3}}, 0\right)}{(Z_{C_V}, \theta_{C_V})}\right) + \left(\frac{\left(\frac{V_{C_T}}{\sqrt{3}}, 0\right)}{(Z_{M_P}, \theta_{M_P})}\right)$$

$$\frac{\overrightarrow{V_F}}{\sqrt{3}} = \left(\frac{V_{C_T}}{\sqrt{3}}, 0\right) + (Z_A, \theta_A)\left(\left(\frac{V_{C_T}}{\sqrt{3}Z_{C_K}}, -\theta_{C_K}\right) + \left(\frac{V_{C_T}}{\sqrt{3}Z_{C_V}}, -\theta_{C_V}\right) + \left(\frac{V_{C_T}}{\sqrt{3}Z_{M_P}}, -\theta_{M_P}\right)\right)$$

$$\frac{\overrightarrow{V_F}}{\sqrt{3}} = \left(\frac{V_{C_T}}{\sqrt{3}}, 0\right) + (Z_A, \theta_A)\left(\frac{V_{C_T}}{\sqrt{3}Z_{C_K}}, -\theta_{C_K}\right) + (Z_A, \theta_A)\left(\frac{V_{C_T}}{\sqrt{3}Z_{C_V}}, -\theta_{C_V}\right) + (Z_A, \theta_A)\left(\frac{V_{C_T}}{\sqrt{3}Z_{M_P}}, -\theta_{M_P}\right)$$

$$\frac{\overrightarrow{V_F}}{\sqrt{3}} = \left(\frac{V_{C_T}}{\sqrt{3}}, 0\right) + \left(\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_K}}, (\theta_A - \theta_{C_K})\right) + \left(\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}, (\theta_A - \theta_{C_V})\right) + + \left(\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}, (\theta_A - \theta_{M_P})\right)$$

In the form of complexes, we have:

$$\frac{\overrightarrow{V_F}}{\sqrt{3}} = \frac{V_{C_T}}{\sqrt{3}} + j0 + \frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_K}}\cos(\theta_A - \theta_{C_K}) + j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_K}}\mathrm{sen}(\theta_A - \theta_{C_K}) + \frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}\cos(\theta_A - \theta_{C_V}) +$$

$$+j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}\text{sen}(\theta_A-\theta_{C_V})+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}\cos(\theta_A-\theta_{M_P})++j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}\text{sen}(\theta_A-\theta_{M_P})$$

$$\frac{\overrightarrow{V_F}}{\sqrt{3}}=\frac{V_{C_T}}{\sqrt{3}}+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_K}}\cos(\theta_A-\theta_{C_K})+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}\cos(\theta_A-\theta_{C_V})+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}\cos(\theta_A-\theta_{M_P})+$$

$$+j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_K}}\text{sen}(\theta_A-\theta_{C_K})+j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}\text{sen}(\theta_A-\theta_{C_V})+j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}\text{sen}(\theta_A-\theta_{M_P})$$

Replacing

$$Z_{C_K}=\frac{{V_{C_T}}^2}{P_{C_{Kn}}}$$

$$\frac{\overrightarrow{V_F}}{\sqrt{3}}=\frac{V_{C_T}}{\sqrt{3}}+\frac{Z_A V_{C_T}}{\sqrt{3}\frac{{V_{C_T}}^2}{P_{C_{Kn}}}}\cos(\theta_A-\theta_{C_K})+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}\cos(\theta_A-\theta_{C_V})+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}\cos(\theta_A-\theta_{M_P})+$$

$$+j\frac{Z_A V_{C_T}}{\sqrt{3}\frac{{V_{C_T}}^2}{P_{C_{Kn}}}}\text{sen}(\theta_A-\theta_{C_K})+j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}\text{sen}(\theta_A-\theta_{C_V})+j\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}\text{sen}(\theta_A-\theta_{M_P})$$

$$\frac{\overrightarrow{V_F}}{\sqrt{3}}=\left[\frac{V_{C_T}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}\cos(\theta_A-\theta_{C_K})+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}\cos(\theta_A-\theta_{C_V})+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}\cos(\theta_A-\theta_{M_P})\right]+$$

$$+j\left[\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}\text{sen}(\theta_A-\theta_{C_K})+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}\text{sen}(\theta_A-\theta_{C_V})+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}\text{sen}(\theta_A-\theta_{M_P})\right]$$

$$\left|\frac{\overrightarrow{V_F}}{\sqrt{3}}\right|=\left|\frac{V_{C_T}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}\cos(\theta_A-\theta_{C_K})+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}\cos(\theta_A-\theta_{C_V})+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}\cos(\theta_A-\theta_{M_P})\right]+$$

$$+\left[\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}\text{sen}(\theta_A-\theta_{C_K})++\frac{Z_A V_{C_T}}{\sqrt{3}Z_{C_V}}\text{sen}(\theta_A-\theta_{C_V})+\frac{Z_A V_{C_T}}{\sqrt{3}Z_{M_P}}\text{sen}(\theta_A-\theta_{M_P})\right]\Bigg|$$

Finally:

$$\left(\frac{V_F}{\sqrt{3}}\right)^2=\left[\left(1+\frac{Z_A}{Z_{C_V}}\cos(\theta_A-\theta_{C_V})+\frac{Z_A}{Z_{M_P}}\cos(\theta_A-\theta_{M_P})\right)\frac{V_{C_T}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}\cos(\theta_A-\theta_{C_K})\right]^2+$$

$$+\left[\left(\frac{Z_A}{Z_{C_V}}\text{sen}(\theta_A-\theta_{C_V})+\frac{Z_A}{Z_{M_P}}\text{sen}(\theta_A-\theta_{M_P})\right)\frac{V_{C_T}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_A}{\sqrt{3}V_{C_T}}\text{sen}(\theta_A-\theta_{C_K})\right]^2$$

$$\boldsymbol{V_F=\left(\left[\left(1+\left(\frac{\cos(\theta_A-\theta_{C_V})}{Z_{C_V}}+\frac{\cos(\theta_A-\theta_{M_P})}{Z_{M_P}}\right)Z_A\right)\frac{V_{C_T}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_A\cos(\theta_A-\theta_{C_K})}{\sqrt{3}V_{C_T}}\right]^2+\right.}$$

$$\boldsymbol{\left.+\left[\left(\frac{sen(\theta_A-\theta_{C_V})}{Z_{C_V}}+\frac{sen(\theta_A-\theta_{M_P})}{Z_{M_P}}\right)\frac{Z_A V_{C_T}}{\sqrt{3}}+\frac{P_{C_{Kn}}Z_A sen(\theta_A-\theta_{C_K})}{\sqrt{3}V_{C_T}}\right]^2\right)^{\frac{1}{2}}\sqrt{3}}$$

The above formula provides the value of the required voltage of the source, considering the data of the feeders and loads, to meet the minimum voltage defined at the load terminals.

11 - ELABORATION OF EXCEL SPREADSHEETS

The Excel spreadsheets were elaborated based on the calculations developed. The theoretical part is not indispensable for its use, but it is important for any further studies.

Two spreadsheets were elaborated, one to determine the voltage in the load when the value of the voltage in the source is known and the other, to determine the voltage in the source to meet the voltage defined in the load. Each folder contains two spreadsheets, one complete and one simplified. In the complete spreadsheet are indicated the formulas of all the terms used in the calculations and, in the simplified ones, which are identical to the complete ones, only the basic information is visible. However, in simplified spreadsheet, the same fields as the full worksheet are hidden but active.

The use of spreadsheets facilitates the calculations, which would be very laborious to do manually.

Spreadsheets can be used for any voltage values of sources and loads.

As in an installation there may be loads with nominal voltages different from that of the system, there are fields to fill in this information. For example, there may be motors with nominal voltages of 440V and 460V.

The fields with power factors of the loads were also left to be filled with the real data.

The reference made to motor(s) is to allow one to consider a single engine or set of motors, the data of which must be defined. For example, if you want to know exactly the behavior of the system with the simultaneous start of two motors with different characteristics. In this case, the equivalent starting current and power factor must be determined to insert them into the spreadsheet.