Sādhanā (2021)46:135
https://doi.org/10.1007/s12046-021-01662-9

Limit analysis of the retaining wall with relief shelves under static surcharge loading using FEM

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MS received 27 February 2021; revised 24 May 2021; accepted 8 June 2021

Abstract. This article examines the stability of retaining walls with relief shelves compared to the conventional cantilever wall using limit analysis integrating with the finite element method. The performance of such walls subjected to an incremental static uniformly distributed surcharge loading on the backfill surface is accessed in the terms of the magnitude of the surcharge carrying capacity of the walls before the incipient failure of the overall structure. From the analysis, it is noted that the provision of an appropriate number of relief shelves and their suitable locations and widths to the wall can bring significant improvement in the stability of the retaining wall compared to a wall without a relief shelf. Furthermore, it is noted that the width and location of the relief shelf are the major governing factors that are susceptible to modify the shape of the potential failure plane, the failure mode of the wall, and the backfill surface settlement profile. To maximize the stability of the wall subjected to static loading on the backfill surface, recommendations are laid for the optimum location and width of the relief shelves for a given height of the wall.

Keywords. Finite element method; potential failure plane; retaining wall; relief shelf; surcharge loading.

1. Introduction

Earth-retaining structures are omnipresent in the man-made environment, which is typically required to provide a support system to any vertical or near-vertical face of the natural and artificial ground for any desired construction [1]. These structures must be designed to withstand the lateral pressures from the retained material and any likely loads from nearby pre-existing structures, surcharge pressures caused due to movement of vehicular traffic, and man-made or natural seismic events.

Extensively implemented conventional rigid retaining wall, such as cantilever type of walls is a cost-effective solution for the retained heights up to 6-8 m, and counterfort or buttress walls can take over a height of 8-12 m as cost-effective earth retention [1–3]. In high cantilever retaining walls, safety against the sliding mode of failure often requires a widening of the base or the development of the shear key under the base. Widening of the base slab carries a significant financial burden on the overall project, while the shear key is fully efficient only in very firm soil to provide additional resistance against the sliding [4]. Also, space limitations often leave no room for the required base width. As a cantilever wall is enlarged in dimensions, it becomes less flexible, and the lateral pressures exerted on it by the soil tend to be higher than the active earth pressure values assumed in the design. If a wall is strengthened to withstand increased lateral pressures, then its rigidity is increased, and the lateral pressures escalate, and so on [5].

In the last few decades, reinforced soil walls are gaining popularity due to their unique flexible nature of the overall structure, which allows them to compensate for any probable differential settlement. For the construction of such walls, a highly essential requirement is a well-graded granular material as these granular materials possess higher shear resistance and good soil-reinforcement interaction. Although, granular material with even a small fraction of fines is not recommended as backfill material in reinforced soil walls, as due to the presence of such fine material, an undrained condition would prevail in the presence of water [6]. So, the selection of suitable backfill material should be cautiously to confirm its suitability in reinforced soil walls. Moreover, the availability of suitable backfill material at a construction site constrains its use for a particular site. Furthermore, the soil reinforcement technique needs a free long-distance (usually 0.6-1.1 times the height of the wall) behind the wall for the construction purpose, and the space constraints in some conditions limit its adaptability.

In view of the above discussion, with the minimization of the sectional dimensions of the wall, the cost of construction material can be significantly reduced, and the same can only be achieved by reducing the total lateral thrust on the wall. Over the years, numerous useful techniques have been utilized for the reduction of lateral thrust on the wall, such as the use of compressible geo-inclusion in the form of expanded polystyrene (EPS) geofoam [7], glass-fiber inclusion [8], or lightweight fill materials like shredded tire chips, plastic bottles, etc. [9–11]. Out of the above-mentioned techniques, the most popular and widely used one is the use of compressible inclusion, which could reduce structural demand on the rigid retaining wall, providing a cost-effective design for such structures [12, 13]. However, the most limiting constraints of this approach include the drainage concern at the wall-backfill interface due to the impervious nature of the EPS geofoam and the accumulated compression of EPS geofoam with time, i.e., creep.

To minimize the thrust on the wall, a way of dealing with the problem is the provision of one or more relief shelves (horizontal reinforced concrete slabs or platforms) of the finite width placed within the backfill and rigidly connected to the wall stem [14]. A relief shelf (pressure relief shelf/relieving platform/shelf) is a thin horizontal cantilever platform of finite width constructed monolithically with the stem of the retaining wall. These relief shelves extend into the backfill parallel to the base slab, provisioned along the height of the wall at suitable intervals throughout the length of the retaining wall as shown in figure 1. The approach of using relief shelf to the wall for the reduction of lateral earth pressure may offer a viable solution, as noted by the many previous studies [15–19], in many countries around the globe [20–25], for a sustainable and cost-effective solution for the retaining structures.

The mechanism of retaining wall with relief shelf is unique and entirely different from the other methods available to reduce the lateral earth pressure, where a relief shelf carries the weight of the backfill above it (up as far as the next relief shelf if multiple relief shelves are provided) and applied surcharge loading if any, and reduces the vertical pressure in the soil lying beneath it. Subsequently, lateral earth pressures on the wall section below the relief shelf get diminished. Concurrently, the vertical force working on the relief shelf exerts a cantilever moment onto the wall stem in the opposite direction to the bending moment caused by the lateral thrust on the wall i.e., the reduced bending moment at the wall compared to the wall without relief shelves [2, 18, 26]; and subsequently decreasing the structural demands of such retaining structures with substantially increased stability (i.e overall factor of safety) [24, 27]. As the appropriate determination of the width and location of the relief shelf is a very crucial engineering judgment to maximize the benefit from the provision of the relief shelf since the short relief shelves could not contribute towards the earth pressure reduction on the wall and wide relief shelf may lead to the overturning of the wall towards the backfill [28].

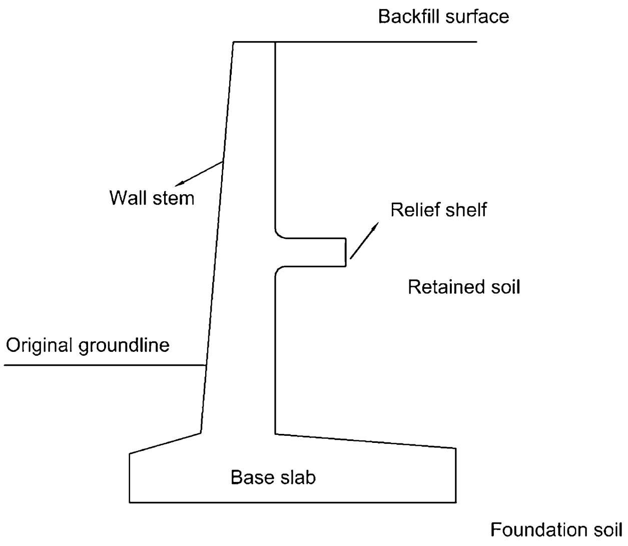

Figure 1. A typical sectional view of a retaining wall with a relief shelf.

2. Objective of the present study

In general, the stability aspect of the retaining wall with relief shelves has not received attention in the available literature, and the full potential of this technique to improve the stability of the wall under static surcharge loading has not been proven very well. There has been an increased recognition that more attention needs to be paid to this area. So, it becomes of utmost importance to have an insight into the stability of such walls and to examine the potential failure modes of the wall with relief shelves under the influence of various governing parameters of the relief shelves. In view of the above, this study aims to bring out an in-depth understanding of the stability aspect of the retaining wall with relief shelves under static surcharge loading with the consideration of various geometrical parameters of the relief shelves and their effects on the performance of wall using the numerical modeling, and further portray the recommendations for a safer and efficient design of such walls.

3. Numerical modeling

To attain the objective of the present study, numerical modeling has been employed in the present work to study the behavior of retaining walls with relief shelves under static surcharge load distributed uniformly on the backfill surface using a computational tool, OPTUM G2 [29] based on the finite element method.

A convenient framework for the numerical modeling to carry out the investigation has been developed based on the earlier studies [15, 25]; which are carrying the concept that for a given height of the wall, a retaining wall with a relief shelf can sustain the higher lateral thrust induced by the backfill and applied static surcharge compared to that of a wall without relief shelf. Given the above fact, the stability of a retaining wall is explored in terms of the uniformly distributed static surcharge carrying capacity of the wall at the backfill surface before the collapse of the overall earth retention system.

A numerical analysis is carried out for a 10 m (very high) height of the wall, *H* to evaluate the effectiveness of the retaining wall with relief shelves to support the uniformly distributed static surcharge at the backfill surface. The dimensions of the wall and backfill are modeled using 6-node Gauss elements as shown in figure 2, where *B* is the width of the relief shelf, and the thickness of relief shelves is taken as 0.4 m, throughout the study. The retaining wall with relief shelves, considered as a rigid one having a unit weight of 24 kN/m^3. Backfill soil having a unit weight of 16 kN/m^3 is considered as dry cohesionless soil (internal friction angle of 35°) and following the Mohr-Coulomb failure criterion. The relative wall-backfill interface movement is controlled by shear joint with interface shear strength defined by the Mohr-Coulomb failure criterion provided at all the places along the face of the wall where soil interacts with the wall [29]. The fixed boundary condition at the bottom of the model and roller boundary condition at the vertical ends of the soil zone are selected to simulate the field conditions of the retaining wall-backfill system. The extent of the applied static surcharge on the backfill surface is ranging from the wall-backfill interface to the end of the backfill such that the surcharge loading is increased evenly till the wall gets failed in both cases, i.e., a wall with and without relief shelves.

The accuracy of the findings of the finite element simulations of various geotechnical problems using OPTUM G2 [29] depends on the selection of the number of elements considered in the finite element mesh as noted from the findings of analysis integrated with either limit analysis method; or the strength reduction method. In the case of limit analysis methods, an appropriate number of elements help to provide an accurate assessment of the stresses and strains generated in the mesh. Furthermore, considering a large number of elements for a given mesh becomes a time-consuming process for the simulation of any geotechnical problem, especially when the size of the considered mesh is enormous. Given above, to figure out the optimum number

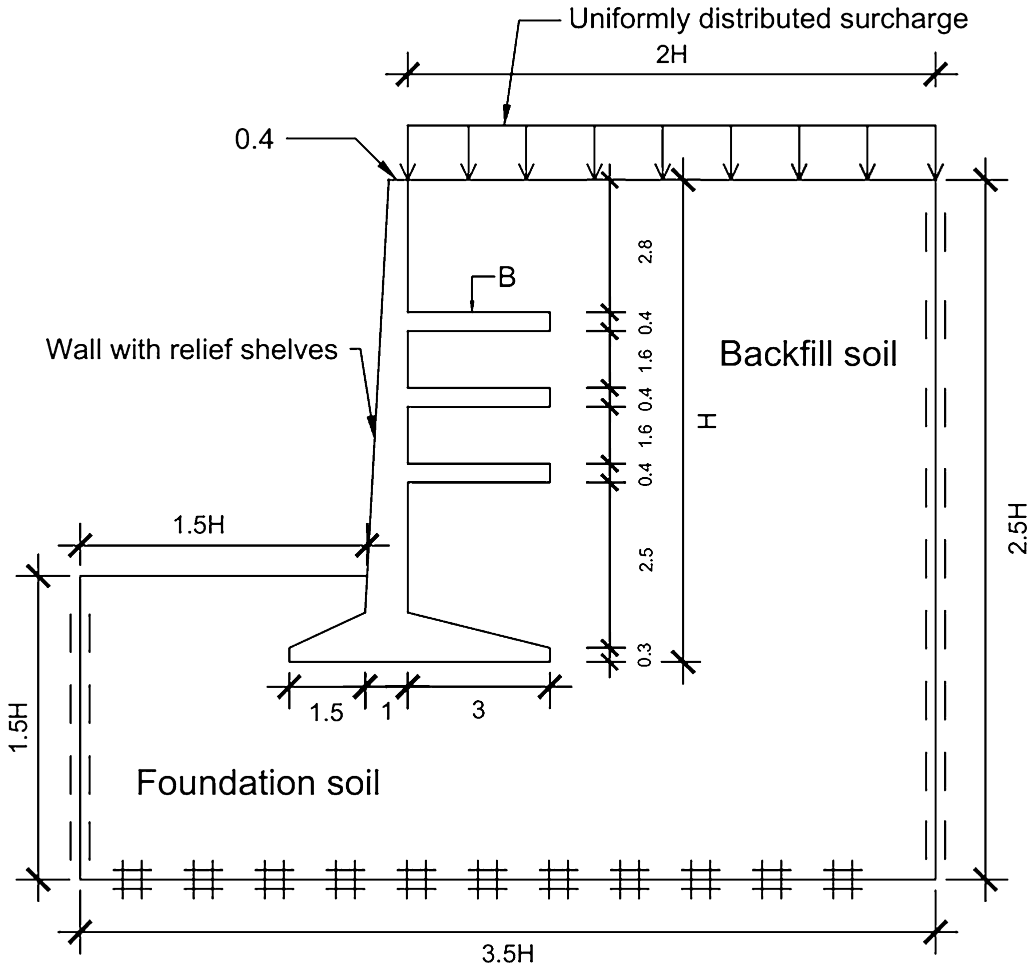

Figure 2. Details of the wall dimensions and boundary conditions of the numerical model considered in the present study (all dimensions are in m).

of total elements in the mesh required for the analysis to obtain precise results quickly, a sensitivity analysis is carried out. In this analysis, the number of elements in the selected numerical mesh is being increased stepwise and the desired outcome is monitored. In the present analysis, the magnitude of distributed surcharge applied on the backfill surface, hereafter, called collapse multiplier load is observed. The variation of the collapse multiplier load (Q_c) and the number of elements in the mesh are shown in figure 3**,** which illustrates that the 5000 elements for the considered numerical mesh are found to be an apt choice to carry out the analysis for the expeditious and precise outcome from the numerical simulation.

To verify the reliability of the present finite element based numerical model, a comparison is made with the findings of a 1-g laboratory model test on a rigid retaining wall with relief shelves, where a 0.6 m high wall with two relief shelves (having 150 mm wider and located at 200 mm and 400 mm depth from the backfill surface) was carried out to study the variation of lateral earth pressure along the height of wall due to the presence of two relief shelves under uniformly distributed static surcharge loading of 50 kPa placed at the backfill surface [26]. Figure 4 shows the simulated numerical model corresponding to the experimental model test [26]. Figure 5 presents a comparison of the earth pressure distributions obtained from the experimental study [26] and the present study for both the walls with and without relief shelves, which shows a good agreement between the present numerical model and the experimental model. Hence, the same model was used to carry out the further parametric studies for the full-scale wall, mentioned in the further section.

A wide-ranging parametric study is carried out by varying the governing parameters which influence the performance of the retaining wall with relief shelves as summarized in table 1. The considered parameters of the wall with relief shelves are the number of relief shelves, the width of the relief shelf (B), and their position along with the height of the wall (z), where z is measured from the backfill surface.

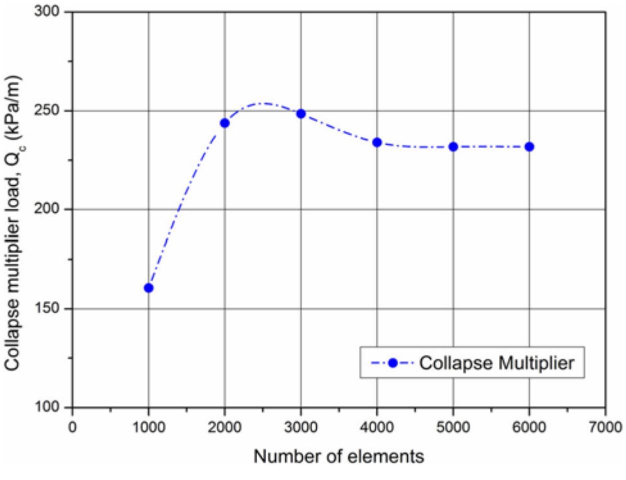

Figure 3. The outcome of the sensitivity analysis for the considered mesh in the present study.

Two descriptions of relief shelves, namely normalized width, β (width factor), and normalized position, α (position factor) of the relief shelves are defined to demonstrate the complete characterization of a retaining wall with relief shelves as well as to ease the interpretation of results obtained from the analysis. The position factor and width factor of a relief shelf are defined as the ratio of the location (z) and width of the relief shelves (B) to the height of the retaining wall (H), respectively. With this convention of notations, a conventional retaining wall without relief shelves is referred to as the wall with β equals zero [28].

To evaluate the ultimate surcharge carrying capacity (collapse multiplier load, Q_c) of a given retaining wall before the incipient failure of the overall wall system, multiplier elastoplastic analysis for the long term is considered in the present work. The multiplier elastoplastic analysis can be recognized as the limit analysis coupled with elastoplastic analysis performed in a step-by-step fashion, where the failure criteria for all solid and structural elements are incorporated with deformations computed at each load step [29]. The soil-wall system is simulated to attain the equilibrium and establish the corresponding initial stress in the overall retaining system. Once the model reaches equilibrium condition, the multiplier static distributed loads are applied on the backfill surface and augmented with unit increment in each step (1 kN/m^2/m) until the retention system collapses while keeping all other loads, i.e., gravity, invariable. The parameters of the relief shelves, normalized width, β (width factor, *B/H*), and normalized position, α (position factor, *z/H*), are varied in the range of 0.1–0.3 and 0.3–0.7, respectively, for the quantitative evaluation of the collapse multiplier load of the retaining wall with relief shelves and the potential mode of the failure of the wall.

4. Results and discussion

This section summarizes and discusses the main findings of the work carried out in the present study. From the numerical simulations, the collapse multiplier load is assessed for the walls without relief shelves and with one, two, and three relief shelves. Collapse multiplier load for the wall with one relief shelf with several combinations of α and β are shown in figure 6, where the magnitude of the collapse multiplier loads are plotted as a function of the width of the relief shelf when the relief shelf is located at various positions ($\alpha = 0.3$, 0.5, and 0.7) along the height of the wall. The comparison of the collapse multiplier for the wall with and without relief shelf reveals that the smaller relief shelf ($\beta = 0.1$) positioned at any location along the wall height is found to be inadequate to improve the

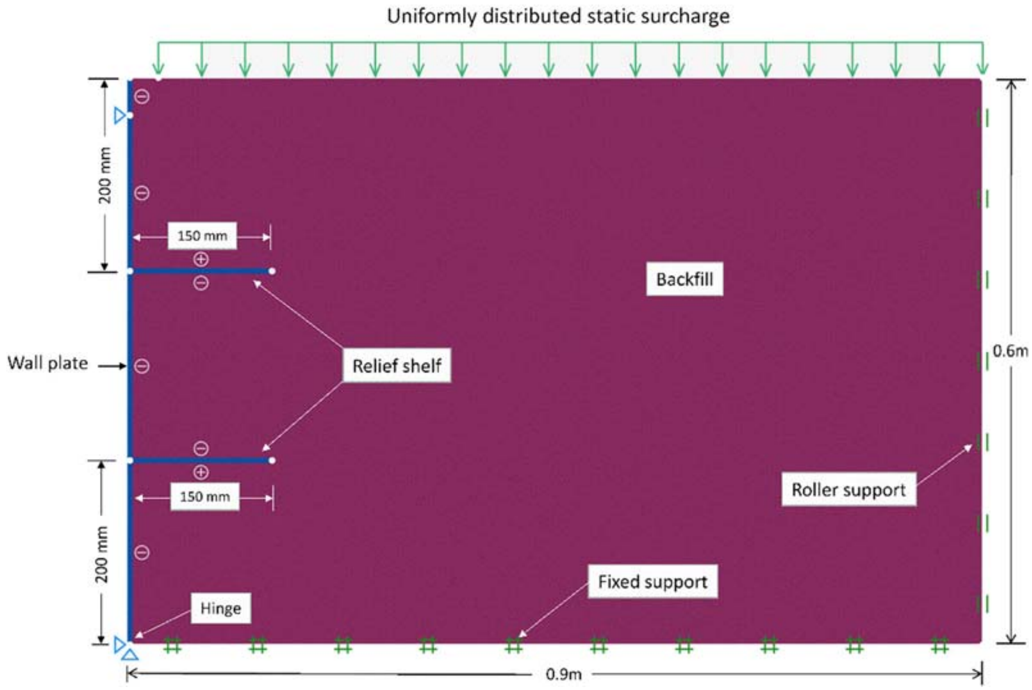

Figure 4. Finite element model using Optum G2 [29] for the validation study.

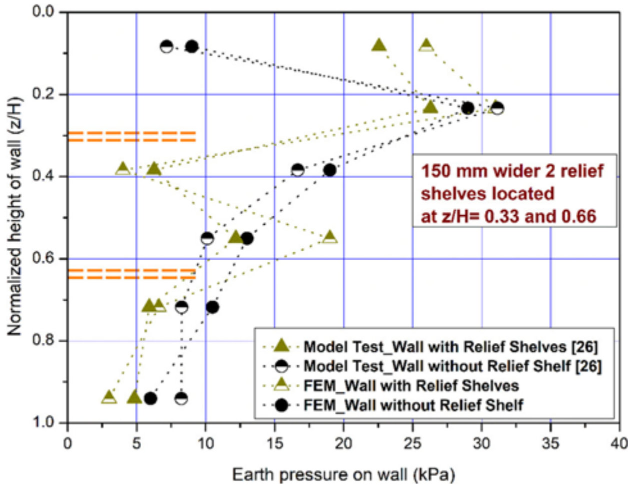

Figure 5. Comparison of earth pressure distribution on walls of the physical model test [26] and the FEM analysis.

collapse multiplier load of the wall significantly. However, with the increase in the width of the relief shelf ($\beta = 0.2$ and 0.3) at any location along the height of the wall, the load-carrying capacity of the retaining wall was remarkably improved. For walls with a single relief shelf having $\beta = 0.2$ and 0.3, the higher stability of the wall (higher the collapse multiplier load is termed as higher stability against the applied surcharge load) was noted in the wall when the relief shelf is located at $\alpha = 0.3$. From the discussion made above, it is notable that the collapse multiplier load increases rapidly when the relief shelf is situated close to the backfill surface/applied surcharge, i.e., $\alpha = 0.3$. Furthermore, the relief shelf located at $\alpha = 0.3$ having width factor, $\beta = 0.3$ could develop the highest stability of the wall among all the cases of the walls with one relief shelf, considered in the present work. The abovementioned observation confirms the fact that a wider relief shelf close to the backfill surface is more efficient than a relief shelf placed at a farther distance from the applied surcharge [28].

A comparison of the shape of the potential failure planes, indicated by concentrations of the shear plastic multiplier, for the walls with a single relief shelf located at $\alpha = 0.3$, 0.5, and 0.7 are shown in figures 7, 8, and 9, respectively. An inclined linear failure plane for a wall without a relief shelf emerges from the end of the heel and meeting at the crest of the wall stem. With the provision of a relief shelf at any height of the wall, the formation of the failure plane gets intervened (whether it can intersect the original failure plane or not) by the presence of a relief shelf, depending upon its width. It further illustrates the fact that the stability of the wall could only be improved by the provision of a relief shelf by a width such that it could intersect the original contour of the failure plane. This, in principle, can give rise to fact that the improvement in the stability of the wall is a result of the formation of a modified failure plane, which contains a larger soil mass in between the failure plane and the wall stem making the wall such that it attracts more static surcharge before it gets failed, i.e., higher stability of the wall. A detailed comparison of the various plastic multiplier plots obtained for walls with a single

Table 1. Details of the parametric study conducted for 10 m wall with relief shelves.

Number of relief shelves	Normalized position of the relief shelf, *z/H* (α)	Normalized width of the relief shelf, *B/H* (β)
1	0.3	0.1, 0.2, 0.3
1	0.5	0.1, 0.2, 0.3
1	0.7	0.1, 0.2, 0.3
2	0.3, 0.5	0.1, 0.2, 0.3
2	0.3, 0.7	0.1, 0.2, 0.3
2	0.5, 0.7	0.1, 0.2, 0.3
3	0.3, 0.5, 0.7	0.1, 0.2, 0.3

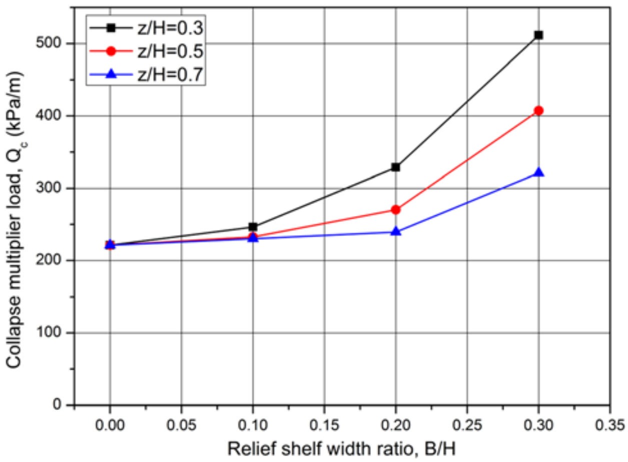

Figure 6. Variation of collapse multiplier of the various wall with a single relief shelf considered in the present study.

relief shelf shows that a shorter relief shelf located near the backfill surface is sufficient to intervene the failure plane of the wall compared to a case when a relief shelf is located at a farther distance from the backfill surface. This also clarifies why the same width of the relief shelf located close to the backfill surface ($\alpha = 0.3$) could maximize the stability of a wall with a relief shelf, compared to a wall having the same width of relief shelf located at $\alpha = 0.5$ or 0.7. Furthermore, the extent of the hindrance to the generation of the original failure plane created by a given width of relief shelf is higher for a wall with a lower α compared to the relief shelf located at a higher α.

The collapse multiplier load for the wall with two relief shelves provisioned with several combinations of α and β are shown in figure 10, where the collapse multiplier loads are shown as a function of the width of the relief shelf, when placed at any two positions out of $\alpha = 0.3$, 0.5, and

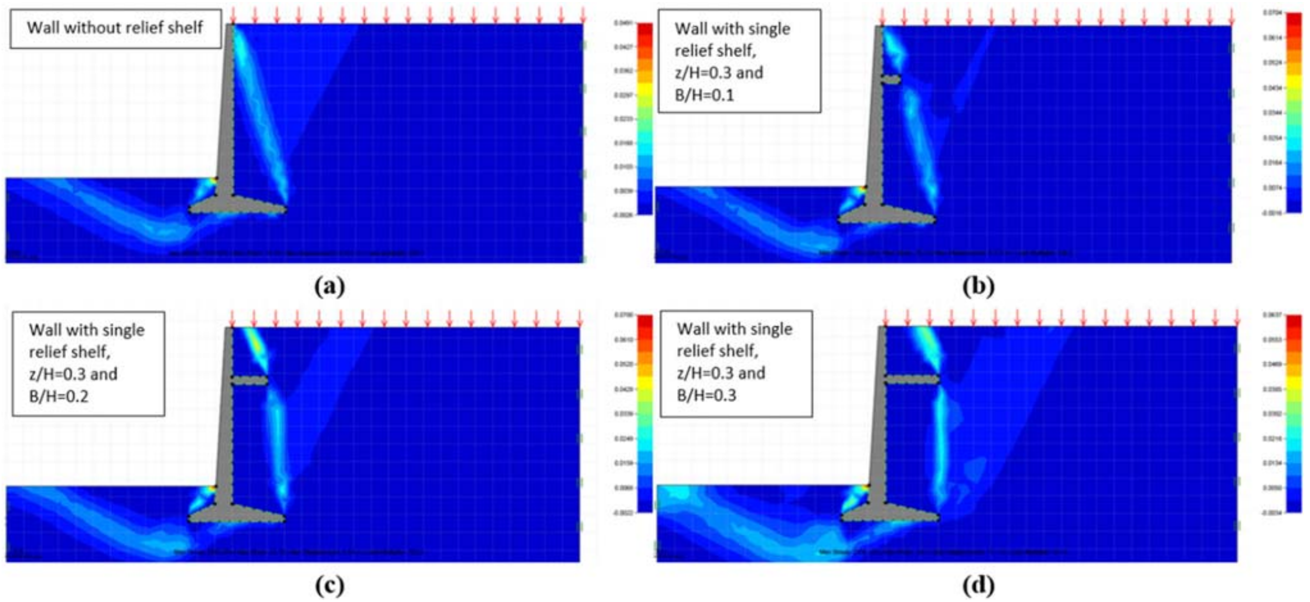

Figure 7. Comparison of plastic multiplier of various walls (a) *z/H* = 0 and *B/H* = 0; (b) *z/H* = 0.3 and *B/H* = 0.1; (c) *z/H* = 0.3 and *B/H* = 0.2; and (d) *z/H* = 0.3 and *B/H* = 0.3.

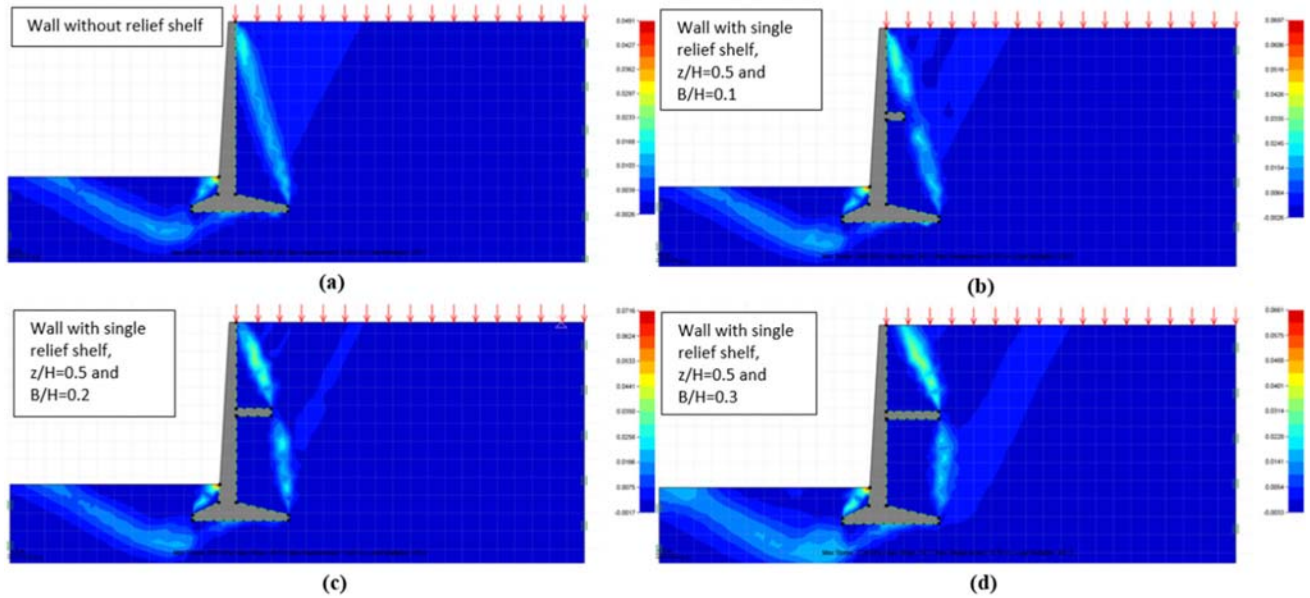

Figure 8. Comparison of plastic multiplier of various walls (a) *z/H* = 0 and *B/H* = 0; (b) *z/H* = 0.5 and *B/H* = 0.1; (c) *z/H* = 0.5 and *B/H* = 0.2; and (d) *z/H* = 0.5 and *B/H* = 0.3.

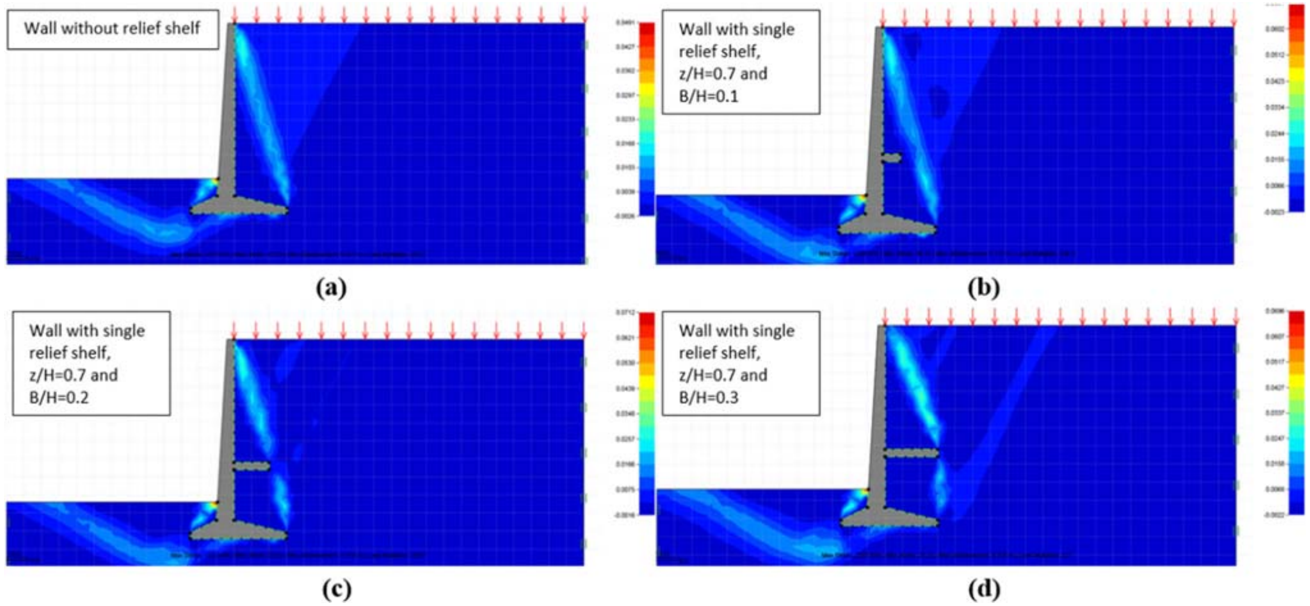

Figure 9. Comparison of plastic multiplier of various walls (a) *z/H* = 0 and *B/H* = 0; (b) *z/H* = 0.7 and *B/H* = 0.1; (c) *z/H* = 0.7 and *B/H* = 0.2; and (d) *z/H* = 0.7 and *B/H* = 0.3.

0.7, having equal width of relief shelves at both positions along the height of the wall.

As seen in figure 10, collapse multiplier load increases with the width of the relief shelf with a given combination of the positions of the relief shelves. Furthermore, the most intriguing finding is that wall with two relief shelves with a given width factor, the maximum collapse multiplier load was noted in the case of the wall when one relief shelf was placed at $\alpha = 0.3$ (close to the backfill surface) out of two relief shelves, i.e., the wall with relief shelves located $\alpha = 0.3$ and 0.5; and $\alpha = 0.3$ and 0.7 compared to the wall with relief shelves located at $\alpha = 0.5$ and 0.7. Moreover, in cases of the wall with two relief shelves having $\alpha = 0.3$ and 0.5; and $\alpha = 0.3$ and 0.7, identical collapse multiplier values were noted.

These observations highlighted the two interesting conclusions. The first one is, when the relief shelf is located at lower α (i.e., close to backfill surface), it majorly

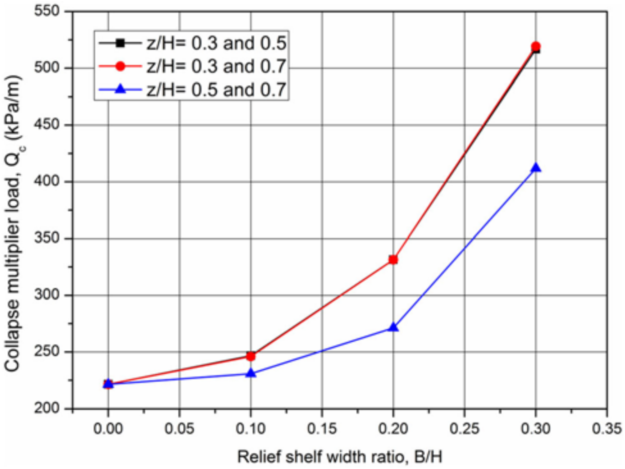

Figure 10. Variation of collapse multiplier of the wall with two relief shelves vs relief shelf width ratio.

contributes towards the stability of the wall compared to a relief shelf placed away from the backfill surface (i.e., higher α). The second observation, which is the most significant finding of this study is that once the one relief shelf is placed at $\alpha = 0.3$, the provision of the second relief shelf at any lower does not affect the collapse multiplier load, as noted in figure 10. The above-noted observations and conclusions are again supported by the comparison of the shape of the potential failure plane for the walls with two relief shelves as shown in figures 11, 12, and 13.

Variation of the collapse multiplier load for the wall with three relief shelves positioned at $\alpha = 0.3$, 0.5, and 0.7 with β ranging from 0.1 to 0.3 and the corresponding potential failure mechanism indicated by concentrations of the plastic multiplier are shown in figures 14 and 15. Moreover, it is noted that even the provision of three short relief shelves ($\beta = 0.1$) is found to be insufficient to improve collapse multiplier load significantly, indicating the fact that the width factor of the relief shelf plays a major vital role when it comes to improving the collapse multiplier load of the wall than the position factor of the relief shelf. This is the most relevant finding and perhaps also the most significant one that the collapse multiplier load is found to be in the range of 511–518 kPa/m for wall with one relief shelf located at $\alpha = 0.3$ with $\beta = 0.3$; two relief shelves located at $\alpha = 0.3$ and 0.5 with $\beta = 0.3$; and $\alpha = 0.3$ and 0.7 with $\beta = 0.3$; and three relief shelves located at $\alpha = 0.3$, 0.5 and 0.7 with $\beta = 0.3$. This result demonstrates the adequacy of using only one relief shelves at $\alpha = 0.3$ with $\beta = 0.3$ to enhance the stability of the retaining wall significantly under the static surcharge, based on all studied cases in the present work. The provision of any additional relief shelf does not modify the shape of the potential failure plane or the collapse multiplier load.

To have a comparison of efficiencies of various wall combinations of the relief shelf's parameters α and β, stability factor, I_q has been coined, which is defined as the ratio of collapse multiplier load of the wall with relief shelf/shelves and without relief shelf. Table 2 summarizes the comparison of the stability factors for all the walls studied in the present study.

A summary of stability factors illustrates the fact that the I_q does not get a considerable improvement of the wall with a smaller width of relief shelf ($\beta = 0.1$) provisioned at any location or in any number along with the height of the

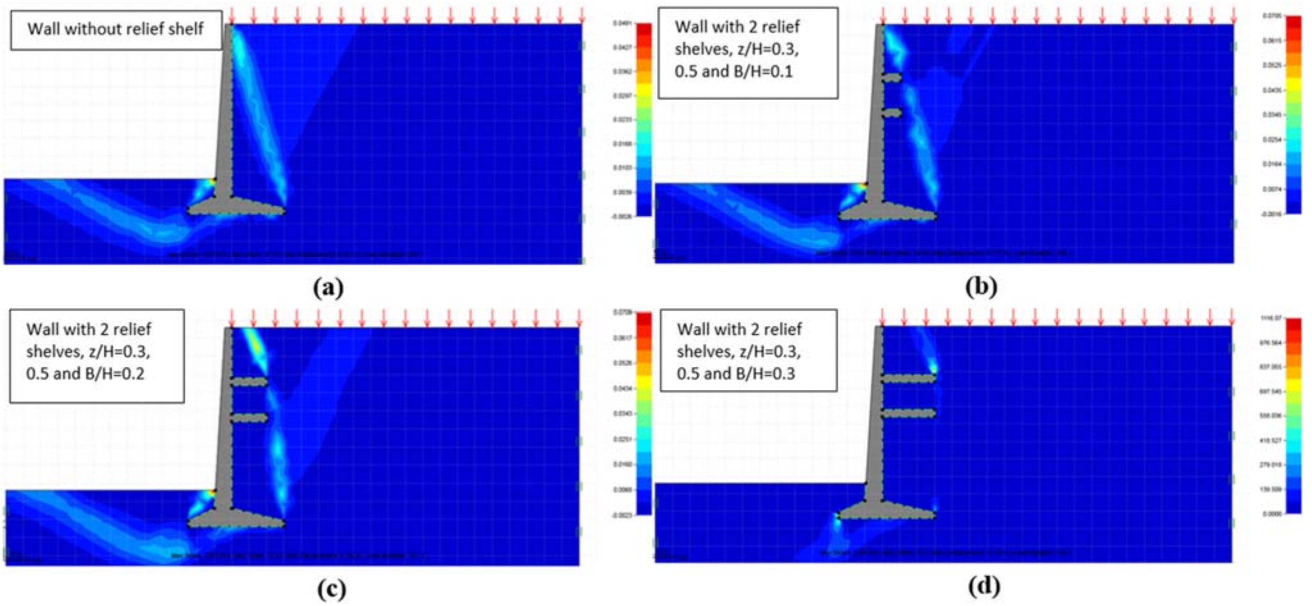

Figure 11. Comparison of plastic multiplier of various walls (a) *z/H* = 0 and *B/H* = 0; (b) *z/H* = 0.3 and 0.5 with *B/H* = 0.1; (c) *z/H* = 0.3 and 0.5 with *B/H* = 0.2; and (d) *z/H* = 0.3 and 0.5 with *B/H* = 0.3.

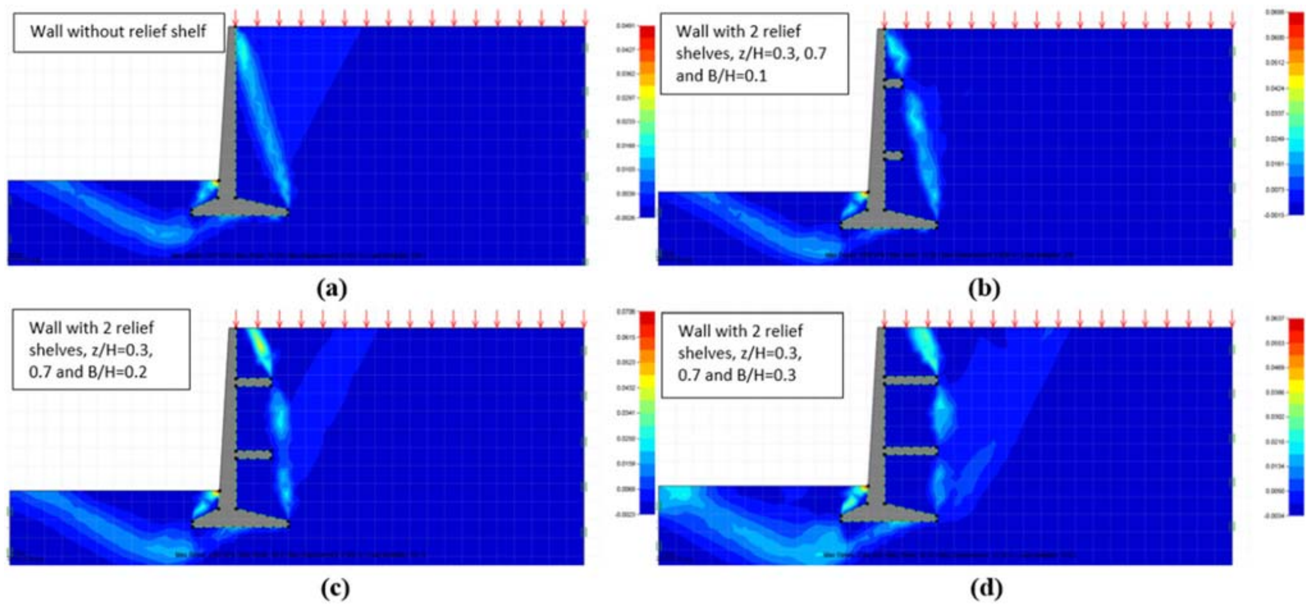

Figure 12. Comparison of plastic multiplier of various walls (a) $z/H = 0$ and $B/H = 0$; (b) $z/H = 0.3$ and 0.7 with $B/H = 0.1$; (c) $z/H = 0.3$ and 0.7 with $B/H = 0.2$; and (d) $z/H = 0.3$ and 0.7 with $B/H = 0.3$.

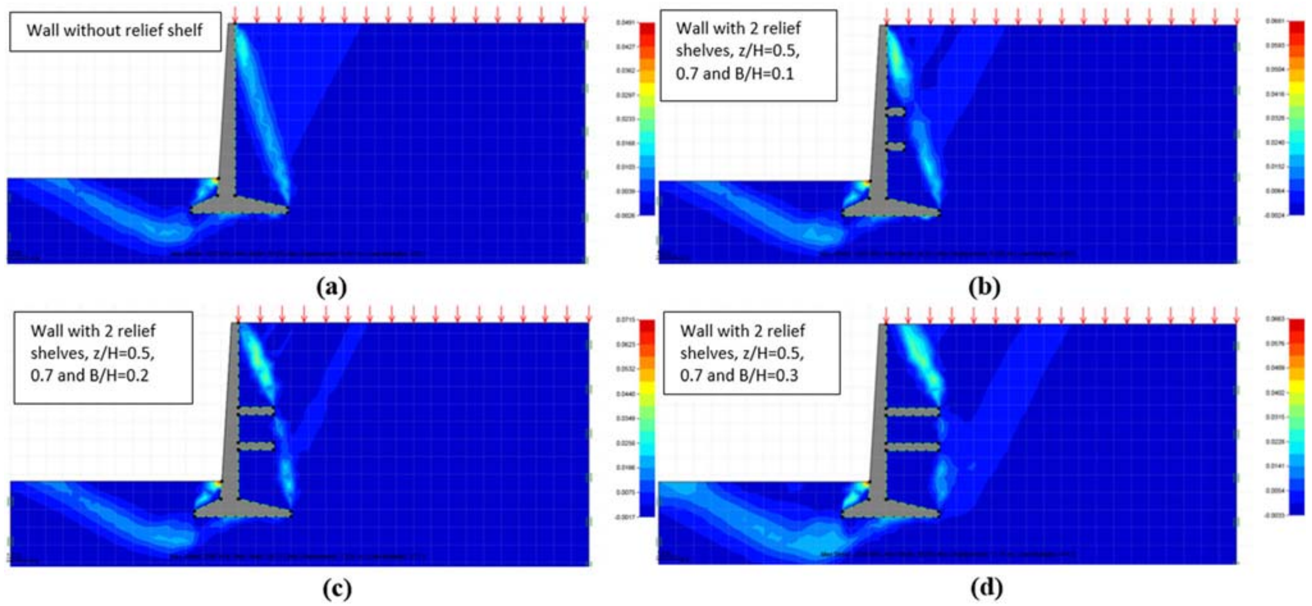

Figure 13. Comparison of plastic multiplier of various walls (a) $z/H = 0$ and $B/H = 0$; (b) $z/H = 0.5$ and 0.7 with $B/H = 0.1$; (c) $z/H = 0.5$ and 0.7 with $B/H = 0.2$; and (d) $z/H = 0.5$ and 0.7 with $B/H = 0.3$.

wall. For the walls with a single relief shelf or multiple relief shelves, i.e., two and three relief shelves, I_q ranges from 1.04 to 1.11. However, it indicates that the relief shelf with a lower value of α (relief shelf located near the backfill surface) proves to be most efficient to improve the stability of the wall against the applied surcharge load.

In all these above-mentioned walls with two or three relief shelves, I_q is found to be 1.11 only, irrespective of the number and positions of relief shelves. These observations provide insight into mechanisms leading to an increase in the overall stability of the wall with relief shelf that as the substantial portion of lateral earth pressure distribution induced by applied surcharge is confined in the upper part of the wall and the same could be reduced only by using the appropriate width of relief shelf in the upper part of the wall if required. This argument is consistent with the findings of [28].

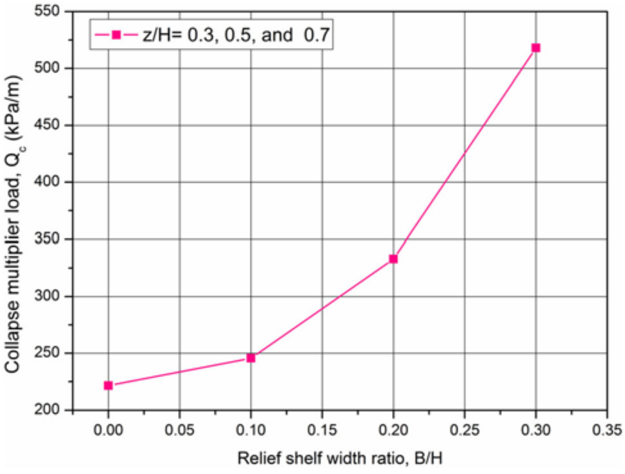

Figure 14. Variation of collapse multiplier load of the wall having three relief shelves.

Similar to the previous trend, all wall with relief shelves combination where relief shelves are placed at $\alpha = 0.3$ have shown higher stability than any case, as one can see I_q ranges from 1.49 to 1.50 compared to all cases of the wall where no relief shelf are located at $\alpha = 0.3$ as in the case of the wall with two relief shelves located at $\alpha = 0.5$ and 0.7, where lowest I_q is noted among all combination of the wall with relief shelves having $\beta = 0.2$.

For wider relief shelves ($\beta = 0.3$), the stability of the wall against the failure under static surcharge loading has increased remarkably in all the cases. However, this trend is noteworthy for a wall with a single relief shelf located at $\alpha = 0.3$, the wall with two relief shelves located at $\alpha = 0.3$ and 0.7; and $\alpha = 0.3$ and 0.5; and wall with three relief shelves located at $\alpha = 0.3$, 0.5, and 0.7, strengthening the fact that the presence of a relief shelf at the topmost location ($\alpha = 0.3$) has resulted in a significant increase in the

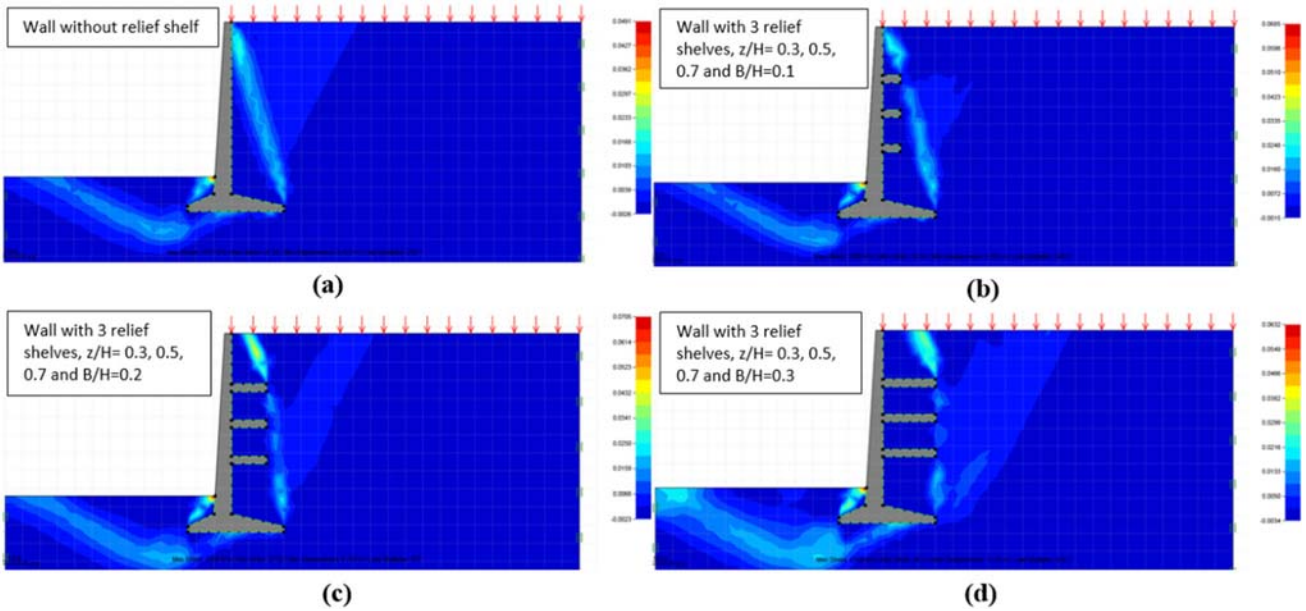

Figure 15. Comparison of plastic multiplier of various walls (a) *z/H* = 0 and *B/H* = 0; (b) *z/H*= 0.3, 0.5 and 0.7 with *B/H* = 0.1; (c) *z/H* = 0.3, 0.5 and 0.7 with *B/H* = 0.2; and (d) *z/H* = 0.3, 0.5 and 0.7 with *B/H* = 0.3.

Table 2. Summary of the stability factors of all the walls with relief shelves.

	Width factor of the relief shelf		
Details of the retaining wall	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.3$
Wall with one relief shelf, $\alpha = 0.3$	1.11	1.49	2.31
Wall with one relief shelf, $\alpha = 0.5$	1.05	1.22	1.84
Wall with one relief shelf, $\alpha = 0.7$	1.04	1.08	1.45
Wall with two relief shelves, $\alpha = 0.3$ and 0.5	1.11	1.50	2.33
Wall with two relief shelves, $\alpha = 0.3$ and 0.7	1.11	1.50	2.35
Wall with two relief shelves, $\alpha = 0.5$ and 0.7	1.04	1.23	1.86
Wall with three relief shelves, $\alpha = 0.3$, 0.5, and 0.7	1.11	1.50	2.34

overall stability of the wall, as discussed in the previous section. It is interesting to note that the stability factor of the wall with one, two, and three relief shelves are almost in the same range of 2.31–2.35. With an objective of the economy of the overall structure, it can certainly be recommended to use a single relief shelf close to applied static surcharge with sufficient width.

In the course of this work, it is discovered that the single relief shelf located near the backfill surface is adequate to increase the overall stability of the retaining wall effectively. Using the insight gained from the outcomes mentioned above and to further investigate the stability aspect of the retaining wall with a relief shelf, the parametric study has been further extended. To maximize the stability of the wall economically, that is the wall with a single relief shelf located at $\alpha = 0.3$ (most advantageous location of the relief shelf to the wall), a parametric study with a single relief shelf conducted by varying β (with $\alpha = 0.3$) ranging from 0.1 to 0.7 is conducted while aiming to investigate the effect of the width of relief shelf on the overall stability of the wall and the transition of the potential mode of failure of the wall with relief shelf.

The trend of the obtained I_q, as shown in figure 16, it is noted that for the short relief shelf, β = 0.1–0.2, I_q is ranging from 1.11 to 1.49, and with further increase in width factor, that is β ranging from 0.3 to 0.5, I_q increases expeditiously from 2.31 to 4.70, however with further change in the width factor of the relief shelf beyond 0.5, increase in I_q is negligible (3.2%). A slight decrease in I_q is noted when β is further increased from 0.6 to 0.7.

To study the wall movement under surcharge loading and possible mode of failure of such walls, a comparison of the plastic multiplier, wall movement, and displacement contour of the soil-wall system for all the wall with a single relief shelf located at $\alpha = 0.3$ is compared with the wall without relief shelf and shown in figures 17, 18 and 19.

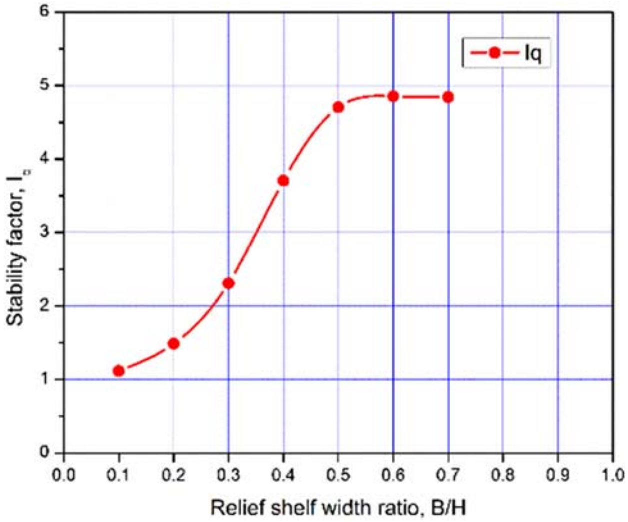

Figure 16. Variation of the stability factor for the wall with one relief shelf vs relief shelf width ratio.

When the retained cohesionless soil mass behind the wall is under an active state of plastic equilibrium, which is resulting due to the lateral strain (stretching in nature) in the soil mass induced due to wall movement, two potential failure planes are being generated termed as the conjugate rupture planes (fracture plane/failure plane) are formed. These rupture planes are further designated as the inner failure plane and outer failure plane. The outer failure plane is close to the wall stem and the inner failure plane is the one that is away from the wall [30].

In figure 17, a representation of the plastic multiplier of the various walls with a single relief shelf placed at $\alpha = 0.3$ is compared with the wall without a relief shelf case. In the case of the wall without relief shelf, a clear outer potential failure plane near the wall stem is observed, originated from the heel of the base slab such that meeting at the crest of the wall stem as an inclined straight line, however, the development of inner failure plane is not so clear, although a small trace could be observed with the slight change in the color contour of the plastic multiplier as shown in figure 17. In the case of the wall with short relief shelf i.e., $\beta = 0.1$, the width of the relief shelf at this position ($\alpha = 0.3$) appears such that it just touches the outer failure plane without altering its shape, thus, no modification in the potential failure plane is noted as expected in case of short relief shelf. This is the reason why the short relief shelf neither alters the lateral earth pressure distribution on the wall nor the overall stability of the wall much. As the width of the relief shelf increases $\beta \geq 0.2$ located at $\alpha = 0.3$, the outer failure plane being intruded by the relief shelf such that its shape is getting deviated from its original position in such a way that it ends somewhere at the backfill surface instead of the crest of the wall stem, as noted in the wall with no relief shelf. The outer failure plane intersects the backfill surface far away from the wall crest as the width of the relief shelf increases up to $\beta \leq 0.5$. Once the relief shelf having $\beta > 0.5$, is used, a circular potential failure plane having one end far behind the retaining wall covering the enormous soil mass behind the wall as well as below the base slab of wall exiting at the non-backfill side of the wall.

The displacement contour of the soil-wall system is schematically shown in figure 18, where the original position of the wall is superimposed with the displaced soil-wall system, to provide an insight into the wall movement and potential failure mode of the walls. It has been observed from the various field monitoring data of retaining wall that the wall movement is neither the pure translational nor the pure rotational one, since it is always the mixed one and dominated by either sliding or overturning, as noted in the case of the wall without relief shelf. Based on the findings of the present study, the general nature of the failure mode of the wall without a relief shelf is found to be an overturning one (about the base slab), such that the wall stem moving away from the backfill and the base slab moves

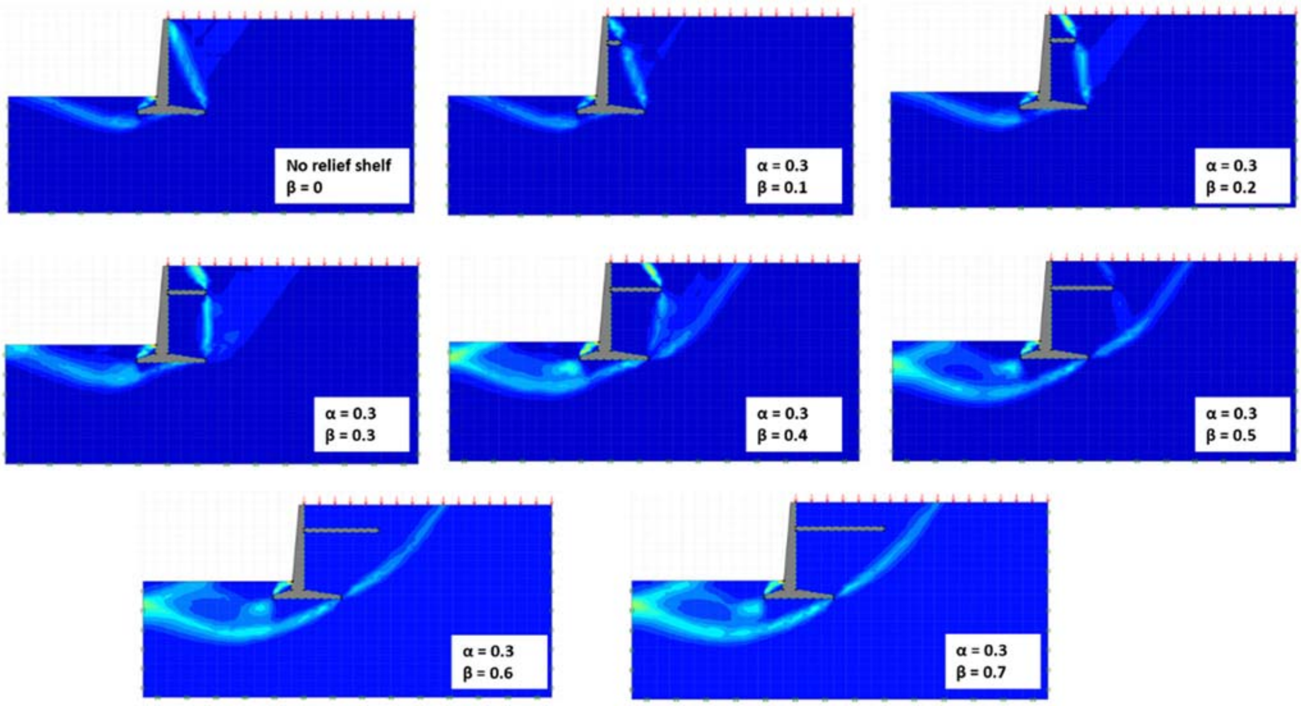

Figure 17. Comparison of the failure mechanism indicated by concentrations of plastic multiplier for walls with a single relief shelf located at $z/H = 0.3$ and without relief shelf.

forward in such a manner that the toe sinks downward and heel rises upwards. Moreover, the movement of the wall away from the backfill is being controlled with the provision of the relief shelf, and the overturning tendency of the wall has been reduced considerably with a great lowering in the outward displacement of the wall stem away from the backfill with the sufficient width of the relief shelf (when $\beta > 0.4$). This behavior of the wall with a relief shelf is caused by the inward moment (moment tending to move the wall towards the backfill) set up by the surcharge and the weight of backfill soil carried by the relief shelf. This moment caused by the presence of a relief shelf is proportionate to the width of the relief shelf and increases with the increase in width of the relief shelf, similar to the findings of the previous investigations [26]. With further increase in width of the relief shelf ($\beta = 0.5$), wall movement was found to be a pure translation one i.e., the wall with relief shelf does not show any overturning except sliding about the base slab. This tendency of the wall movement arises from the fact that the moment generated by the provision of the sufficient width of relief shelf is counterbalanced by the outward moment (moment tending to move the wall away from backfill) as noted in the previous studies [25, 26]. However, with further increase in the width of the relief shelf, $\beta > 0.5$, the inward moment dominates such that it makes the wall slide and overturn towards the backfill when $\beta \geq 0.6$, which is reversed behavior to the noted behavior of the wall having β ranging from 0.0 to 0.4.

In the present analysis, the wall is considered as the rigid one, and the same is validated from figure 18, where no distortion in the shape of the wall is noted, i.e. the angle between the relief shelf and wall stem remains right angle even after the wall movement. This observation is consistent with the former study's assumption [25] used for establishing the analytical framework using the static force analysis of the retaining wall with relief shelves. Furthermore, this wall movement is accompanied by the heaving of the soil placed at the toe. Observed heaving in case of the wall without relief shelf is such that it is highest near the toe and subsidies as one moves away from the wall on the non-backfill side. This finding is obvious since as the wall stem moves away from the backfill, soil near the junction of wall stem and base slab on the non-backfill side gets pushed and this soil loosening is more or less similar in the case of the wall with relief shelf having $\beta = 0$, 0.1 and 0.2. However, as the width of the relief shelf increase, it creates a tendency for the wall to move towards the backfill. This movement is such that the toe moves upward; and the heaving near the toe takes a transition from the maximum at the toe to the minimum at the toe and the utmost heaving shifts away from the toe as noted in cases of the wall with $\beta > 0.3$, and the effect was more pronounced with higher width of relief shelf, which can also be noted from the displacement vector contours of the wall as shown in figure 19.

The displacement vector contours have demonstrated the most interesting finding that the dominant region of soil

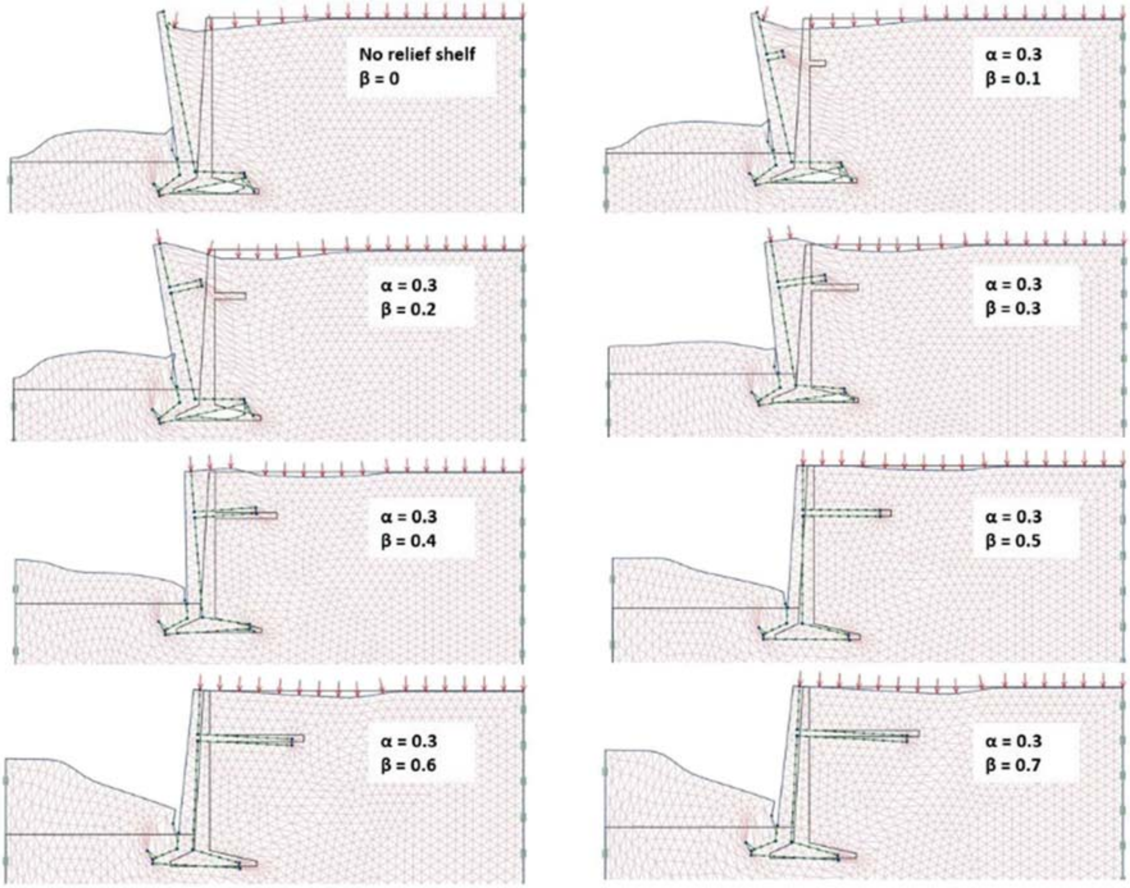

Figure 18. Representation of movement of the walls with a single relief shelf located at *z/H* = 0.3 and without relief shelf.

movement behind the wall in the case of the wall without a relief shelf, contained in an inclined potential failure plane originating from the heel of the base slab. The potential failure plane gets widened without any appreciable change in the shape up to $\beta = 0.3$. However, when $\beta > 0.3$, a curved potential failure plane is observed to be generated such that an enormous soil mass is taking part during the failure along a circular failure plane, as seen in figure 19. The obtained potential failure plane is that a deep-seated circular failure wedge is getting generated below the base slab which signifies the information that the enormous soil mass is taking part during the failure, that is the circular slope failure instead of the wall failure even that is occurring when the applied surcharge is enormous, i.e. almost five times of the surcharge load sustained compared to the wall without relief shelf before it fails.

With rigorous examination, it is noted that the provision of relief shelf dominantly contributes to the formation and the shape of critical wedge behind the wall such that the wedge makes an angle of 72°, 66°, 62°, 60°, and 54° with the horizontal (measure at the heel) for the wall with a relief shelf having $\beta = 0$, 0.1, 0.2, 0.3 and 0.4, respectively (figure 19). Despite the large changes in critical failure angle till $\beta = 0.4$, a relatively small difference in the failure angle is noted with a further increase in the width factor of the relief shelf. The potential failure plane does not change its shape much and the failure angle remains constant equal to 52° for all walls with $\beta \geq 0.5$.

The transition in the mode of wall movement due to the increased width of the relief shelf has not only affected the soil heaving and the failure plane behind the wall but also governed the extent of the zone below the base slab where the dominant movement of the soil mass is observed. This noted behavior is reflected in terms of the zone of displacement vector contours of the soil below the base slab as seen in figure 19, where *D* is the extent of the soil zone below the base slab where major displacement vector contours are noted. The extent of this distance, *D* is found to be 3.87m for the wall without a relief shelf, which is almost 0.7 times the base slab width. The above-mentioned depth of soil zone increases with the width of the relief shelf, as 3.95m and 4.22m for the wall with relief shelf having $\beta = 0.1$ and 0.2. However, this change is found to increase exponentially from 4.82 to 6.81m with further change in β from 0.3 to 0.5 and attains a constant value of 6.88m for the walls with $\beta \geq 0.6$. The change in soil zone

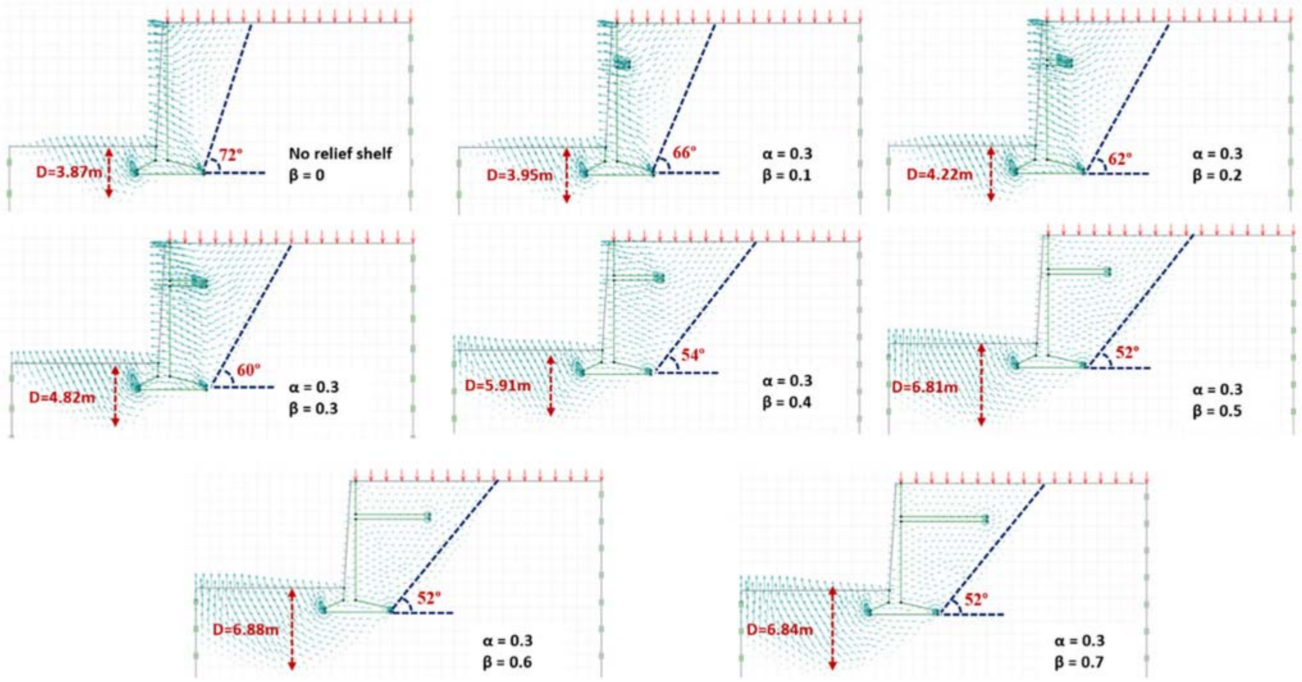

Figure 19. Representation of displacement vector contours and potential failure planes of the walls with a single relief shelf located at $z/H = 0.3$ and without relief shelf.

under dominant displacement is due to two factors, namely the mode of wall movement, and the collapse multiplier load. The trend of D, is similar to the trend of the magnitude of collapse multiplier load, as both increase linearly initially till $\beta \leq 0.2$, and shoots up in the range of $0.3 \leq \beta \leq 0.5$ and further achieves a constant value for wall with $\beta \geq 0.6$.

Backfill surface settlement is also an important consideration to take care of the serviceability criterion as the higher backfill surface settlement may lead to a potential risk to the stability of the structures resting on or nearby the backfill surface. In the case of the wall without a relief shelf, backfill surface settlement is high near the wall stem as the wall moves away from the backfill. The most significant observation of this study is that, with the addition of a relief shelf to the wall, the backfill surface settlement contour changes its shape drastically by lowering the backfill surface settlement near the wall stem. This is largely attributed to the fact that the presence of a relief shelf to the wall acts as a rigid barrier that does not allow the backfill to settle down behind the wall up to the extent of the width of the relief shelf. This behavior is more pronounced in the case of the wall with $\beta \geq 0.3$, where the backfill surface is almost horizontal behind the wall up to the extent of the width of the relief shelf. The profile of the backfill surface settlement is almost a straight line up to the length equal to the width of the relief shelf behind the wall and a shallow curvature takes the shape of the backfill surface beyond the influence of the relief shelf. However, as the wall moves forward, the soil behind the walls slides downward along the wall face, resulting in the backfill surface settlement. However, the trend of the maximum backfill surface settlement is such that it decreases with an increase in β and gets minimum at $\beta = 0.5$. In the case of a wall with $\beta = 0.5$, only a slight movement of the wall in the forward direction occurs due to the forward sliding of the base slab, as discussed earlier. With further increase in the width of the relief shelf, $\beta \geq 0.6$; backfill surface settlement increases, which is caused by the downward movement of the wider relief shelf from its original position into the backfill.

5. Conclusions

A comprehensive numerical study is carried out for a cantilever retaining wall with and without relief shelves subjected to incremental uniformly distributed static surcharge loading on the backfill surface. The stability of the wall is assessed in terms of the magnitude of the surcharge carrying capacity of the wall before the incipient failure. For a 10 m height (H) of a wall, three positions (z), measured from the backfill surface, were selected for positioning the relief shelves i.e., $z/H = 0.3$, 0.5, and 0.7 with the various combination of relief shelves in terms of their number and width are investigated. It is noted that with the appropriate selection of relief shelf parameters, like their number, location, and width, a retaining wall with a relief shelf can be a potential solution for a stable earth retention system. In the light of reported results of the present study, it is conceivable that the same technology can be use used for providing a sustainable solution for the earth retention of side slopes of the embankment subjected to a possible heavy static surcharge in the form of traffic loading induced from road/rail track. The results of this work unravel and shed light on the understanding of retaining walls with a relief shelf. Based on the outcome obtained from the present study, the main achievements, including contributions may be summarized as follows:

1. The provision of the relief shelf to the wall can lead to significant improvement in the stability of the retaining wall to carry the static surcharge loading on the backfill surface compared to a wall without a relief shelf. From the present study, it is established that the wall with a relief shelf (with an appropriate location and width) can support almost five times uniformly distributed surcharge loading at the backfill surface before the incipient failure compared to a wall without a relief shelf.
2. To increase the stability of the wall subjected to static loading on the backfill surface, the relief shelf should be placed near the backfill surface. From the work carried out in this study, it is noted that $z/H = 0.3$ is the optimum position for the provision of a relief shelf.
3. Relief shelves are susceptible to modify the wall movement of the retaining wall depending upon its width and location along the height of the wall. As the width of the relief shelf increases, the tendency of the wall rotation about the base slab away from the backfill shifts towards the wall movement into the backfill. During this transition, a state of the pure translational mode of failure of the wall suggests that for a given configuration of the wall with suitable relief shelf arrangement, outward moments of the thrust acting on the wall can be equilibrated by the inward moment due to the weight of soil and surcharge supported by the relief shelf.

   For the wall studied in the present study, the short relief shelf ($\beta \leq 0.2$) could not modify the failure mode of the wall compared to the wall without a relief shelf. Wall having relief shelf parameter, $0.3 \leq \beta \leq 0.4$ reduces the overturning tendency of the wall about the base significantly. For $\beta = 0.5$, the wall is tending to slide only, and with $\beta \geq 0.6$, the wall tends to overturn about the base slab towards the backfill side.
4. The profile of the backfill surface settlement is significantly governed by the presence of a relief shelf. It is reduced significantly compared to the wall without relief shelf, as well as the location of the maximum backfill surface settlement, which moves far away from the stem of the wall with an increase in the width of the relief shelf.
5. The wall with a single relief shelf having $\alpha = 0.3$ and $\beta = 0.5$ is found to be the optimum configuration to maximize the stability of the wall without negotiating the serviceability criterion i.e., backfill surface settlement.

Acknowledgements

The author gratefully acknowledges the financial support received from the Madan Mohan Malaviya University of Technology, Gorakhpur, Uttar Pradesh, India (Letter No./MUT/Dean.PGSandR&D/RIG/966/2018), to carry out the research work.

Data availability Some or all data, models, or codes that support the findings of this study are available from the corresponding author upon reasonable request.

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